# NSERC 2002 Reallocation Document
# Pure and Applied Mathematics
# GSCs 336/337


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# Introduction

Mathematics in Canada is represented by an energetic and cohesive community with international impact, a very effective infrastructure for the support and dissemination of research (through its 3 mathematical institutes and their impressive initiatives), and a strong involvement in cross−disciplinary activities. In the past decade, Canadian mathematics has undergone a fundamental restructuring and has emerged with a clear awareness of the discipline’s central and pervasive role in Canadian society and of the urgent need to fashion a strategy, built on that reality, for the community. The foundation for this activity is the stable funding of individual researchers and their research groups, the majority of which is provided through GSCs 336/337.

To build on our momentum, we are focusing our goals for the funding proposals on both excellence and the future: namely, on top researchers, tomorrow’s leaders, and new applicants. Our proposals will request the following:

(1) Proposal A: targeted funding for recognized top researchers who can assume a leadership role involving an active focused group;

(2) Proposal B: targeted funding for younger emerging leaders whose research and training activities have moved to a higher level; and

(3) Proposal C: adequate entry grants for the very large number of junior and senior new applicants who will be applying to GSC 336/337 over the next 4 years.

The document is structured as follows: Parts 1 and 2 provide the vision and strategy for Canadian mathematics and the funding proposals for achieving them; Part 3 is a report on the Canadian mathematics community; Part 4 provides an overview of the 3 mathematical institutes; Part 5 is a report on new initiatives in infrastructure; Part 6 addresses the training of highly qualified personnel (HQP); Part 7 provides further considerations on the funding proposals and the consequences of no increased funding; and Part 8 is an appendix containing details concerning each of the mathematical institutes.

# Part 1: A Vision for Mathematics in Canada

### A New Reality

The role and impact of the mathematical sciences within the global scientific, technological, and biomedical enterprise has grown at an astonishing rate over the past decade. The conceptual and computational tools of mathematics have become essential for progress in many areas of the life sciences, information and communications technologies, nanosciences, and financial and industrial sectors. This is a new reality for mathematics. The interdisciplinary nature of mathematics and its "*critical role in advancing interdisciplinary research*" was stressed in the recent NSF (US National Science Foundation) budget statement before the House Appropriations Committee. Mathematics is "*a powerful tool for insight and a common language for science and engineering*." In what it described as "*a centerpiece of NSF’s core investments,*" the NSF proposed to double its funding for mathematical research over the next few years.

Canadian mathematics is fully engaged in this new reality. Over the past 10 years, Canadian mathematicians have forged substantial links to a broad spectrum of scientific disciplines and emerging directions of research, and to financial and technological sectors. There is a need to maintain such momentum. The full participation of Canadian science and technology in this highly mathematical and interdisciplinary world can be achieved only with a strong force of mathematically−literate and mathematically−sophisticated scientists. The training of HQP in mathematics is a high priority.

### A Strong Discipline

Mathematics is a rich and active science with its own internal dynamics and its own sources of fundamental problems and conjectures in addition to being a conceptual framework and source of powerful tools for science and technology. The long−term importance and relevance of mathematics to scientific endeavor as a whole has been shown, over and over again, to be based on the strength and vigour of its own core research. Furthermore, the ability of Canadian mathematics to participate in the greatly expanding role of the mathematical sciences is necessarily founded on the quality of its researchers. The rise of Canadian mathematics to its current level has been a 20−year process, achieved through constant recruitment and development of talented new researchers. The current generation of younger researchers is internationally prominent in many major areas.

### Priorities

The priorities of Canadian mathematics are:

(1) to strengthen its leadership in fundamental and interdisciplinary research, providing crucial mathematical resources for science and technology; and

(2) to increase its capacity to recruit, support, and train HQP in mathematics.

### Mathematical Institutes

The presence of 3 mathematical institutes capable of providing innovation and leadership in research is a major force behind the recent success of Canadian mathematics. The research institutes in the mathematical sciences−Le Centre de Recherches Mathématiques (CRM), the Fields Institute for Research in Mathematical Sciences (Fields), and the Pacific Institute for the Mathematical Sciences (PIMS)−have had a tremendous influence on the Canadian mathematical community. The institutes have organized numerous scientific programs in major areas of current research, with emphasis on outreach, interdisciplinary, and international activities. They have led the way in building bridges between Canadian mathematics and other disciplines, and have acted as major training centres for young talent. The institutes have also pioneered the building of partnerships between Canadian mathematicians and the industrial and financial world. As part of this activity, they brought together researchers from mathematical sciences and industry to form the national network, MITACS, and the Montreal−based network, NCM_2. The most recent institute innovation is the Banff International Research Station (BIRS), a collaboration between PIMS and the Mathematical Sciences Research Institute in Berkeley (through NSERC, the NSF, and the Alberta government). BIRS will provide Canadian mathematicians and users of mathematics with a major forum for research−intense workshops, promoting both mathematical research and its interactions.

The mathematical institutes are a vivid demonstration of the fact that many frontiers of knowledge are concentrated in areas that cut across traditional disciplines, but possess mathematical tools as their common denominator. Mathematics is the only discipline in Canada to have created such large scale (and effective) institutions for interdisciplinary activity.

## Part 2: Strategy and Funding Proposals

The need for increased funding in mathematics is pervasive. In this proposal, however, we limit our requests to the case of certain key groups of researchers. In the last Reallocation Exercise, taking this approach proved highly effective in advancing mathematical research and training.

### Implementation of Previous Proposals

The 1998 Reallocation Committee provided resources to GSCs 336/337 for two categories of researchers: $539K for new applicants and younger researchers, and $323K for top researchers. This infusion of funding has had a significant impact on a large number of researchers and on the training of HQP. The following 3 statistics convey this impact and demonstrate where much of these allocated funds were applied: (i) there was an increase in the number (from 32 to 59) of mathematicians with grants of $30K or larger (i.e., roughly twice the average grant in GSCs 336/337) in the period 1997−2001. In the process, these 59 mathematicians received an extra $570K of funding, as the total value of their grants rose from $1.6M to $2.17M; (ii) the 1998 mathematics submission provided a list of 31 "*younger researchers at the forefront of exciting emerging fields*." In the past 4 years, the total value of the grants of these young researchers rose more than 50% from $503K to $777K; and (iii) providing more money for the support of postdoctoral fellows (PDFs) was an implicit goal of the 1998 funding requests. There has been a striking increase (over 50%) in research grant expenditures on PDFs over the past 3 years (HQP, Table 16), reflecting the additional funding provided for key researchers. Moreover, these expenditures have been leveraged by at least 100%−almost all mathematics PDFs are financed by partnering research funds with other sources.

## Discipline Dynamics

Several dynamics in Canadian mathematics have shaped our perception of the precise funding proposals to be pursued. These discipline dynamics are detailed here.

***(a) Larger Scale Research and Training***: The existence of groups directed towards targeted, very often interdisciplinary, areas of research is familiar to the experimental sciences, but is a new and rapidly expanding phenomenon in mathematics. There are many forces encouraging and shaping such groups, such as:

- the active support of the mathematical institutes or MITACS;
- the growing emphasis on interdisciplinary activity. There is an increasing complexity to research projects having a well−defined mathematical component, resulting in an expanded need for professional expertise and PDF support;
- the surge in university−based labs targeting the private sector; and
- the drive within Canadian universities to define research and training priorities and to establish long−term hiring plans that reflect those priorities.

This phenomenon has had a substantial cumulative effect. Typically, this research format is built around a leader and involves a combination of PDFs, programming and technical assistants, graduate students, university researchers, and visitors. Such groups provide wonderfully stimulating training grounds (and magnets!) for these graduate students and PDFs. This model for the mathematics community is creating a need to finance such leaders at a level beyond that previously available for single individuals.

***(b) Demographic Pressures***: The current turnover in university faculty is producing a rapid increase in the fraction of university mathematicians who are strong active researchers and therefore potential NSERC grantees. This rapidly growing strength of the discipline is reflected in the 10% rise in the number of grantees in GSC 336/337 in the past two years (DD, Table 1). The number of new applicants for the past 3 grant competitions (including the present one) are 47, 47, and 52, respectively; these totals include 12, 13, and 23 senior news, respectively. This demographic surge will continue. For example, 42 tenure track positions for mathematicians are advertised for the coming year; 14 of these are at the level of associate professor or higher. The Canada Research Chairs (CRC) program will also have considerable impact on mathematics departments, and hence on GSCs 336/337, in the next few years. A survey of Canadian mathematics departments yields commitments for 31 CRCs over the next 5 years, evenly spread between Tier I and II; 10 CRC positions are presently being advertised. The current 9 CRCs funded through GSC 336/337 have an average grant of over $41K.

These demographics are wonderful news for Canadian mathematics and its future, but a mixed blessing for GSCs 336/337. An influx of new talent raises serious concern about the ability of these GSCs to provide adequate grants for all of these deserving new applicants over the next few years without seriously under−funding many strong researchers currently receiving grants. The scale of this concern has led us to submit a separate funding proposal for new applicants.

***(c) Increased Funding Needs***: The funding needs of mathematicians are intensifying. As increasing numbers of mathematicians become involved in larger scale research (e.g., leading an active group or directing an expensive project), there is an urgent need for significantly more funding. There is also a collective need for more funds. As already noted, a large number of young talented researchers are entering the system; however, it will be shown (see Part 7) that many excellent researchers currently in the system will not be leaving. Funding an increasing number of active researchers requires more money. In addition, current funding levels in mathematics are too low to adequately support the necessary numbers of HQP. The current average grant of $16,580 (only $2,250 above what it was a decade ago) (DD, Table 3) supports at most one Masters student per year. Even larger grants in mathematics cannot support a full PDF; the importance of PDFs in mathematical research programs will be emphasized in Part 6. For the information of our referees, we point out that university support of HQP in Canada is typically limited to salary provided by sessional teaching and TA duties. Research grants are a necessary and major component of HQP support.

## Three Funding Proposals

We have developed 3 funding proposals based on these current impacts and on future trends in Canadian mathematics. For each, we give the rationale followed by the funding request.

***(a) Leadership Support***: There is a new discipline dynamics in mathematics−group−based research is being carried out on a larger scale than ever before. There are now both very effective established groups and a number of emerging groups, each centered around key individuals. Examples of such groups and their activities are provided in Part 3. Added resources are required for existing groups to realize their full potential and for fledging groups to evolve to a more structured level. The local presence of adequate funding is a primary reason for the existence of the existing groups where they have emerged. Notably, the evolution of group activity is more pronounced in Quebec and western Canada.

It is essential for Canadian mathematics that we support and encourage this new type of dynamic and the leaders who are driving it. To do so, GSCs 336/337 must provide targeted funding to recognized researchers who are assuming, or have assumed, a leadership role involving an active group, typically focused on training or a major research project. Such funds would provide resources to allow researchers to carry out a structured initiative. To have a real impact, at least 20 leaders require targeted funds with an average value of $50K per year, normally awarded for a 4−year period. Important criteria would be demonstrated need and a credible plan for pursuing alternative funding in the case of longer−term activity. Awards could either be part of the normal grant cycle or be dispersed in a special competition.

***Proposal A*: Additional funds to promote structured initiatives by recognized leaders in the community during the period 2003−2007; this requires $50K for each of 20 leaders, amounting to $1M per year.**

***(b) Emerging Leaders***: There is an impressive number of junior researchers in both traditional and emerging areas with high impact and visibility, and who must be funded in proportion to their activity. These rising stars can be identified as grantees seeking their first or second renewal who have a distinguished research record of at least 6−10 years duration. At that stage, a better assessment of quality and financial needs can be made. These emerging leaders also have growing financial needs, particularly with respect to training of HQP.

***Proposal B*: Additional funds to enhance the research and training capabilities of emerging leaders during the period 2003−2007; this requires $10K for each of 60 younger researchers, amounting to $600K**.

***(c) New Applicants***: The demographic statistics presented above strongly suggest that the number of grantees in GSCs 336/337 will increase sharply over the next 4 years and that, on a yearly basis, GSCs 336/337 will need to award 35−40 new grants, including at least 10 to senior new applicants.

The current average grant in mathematics for junior new applicants is $11K. This is a minimal entry grant whose level is ultimately explained by the current low value of mathematics grants. The target should be to award junior newcomers the current average grant in mathematics, namely $16.5K. The activity level of new grantees easily warrants such entry grants. As a comparison, this is almost exactly the average entry grant over the past 4 years in Computing Science (DD, Table 15), which is a group of beginning researchers with similar funding needs. The funding needs of senior new applicants, because of their consistently high level of activity, will average twice (e.g., $33K) that of junior new applicants.

We have also used NSERC's own data to analyze the situation regarding new applicants for GSCs 336/337 (see Part 7, below). The analysis leads to two conclusions:

(1) About 25 new grantees per year will apply to GSC 336/337. This definitely contrasts with the GSC reality of 47, 47, and 52 new applicants in the last 3 competitions. Moreover, NSERC data does not take into account senior new applicants.

(2) The money to become available for redistribution via attrition in GSCs 336/337 will essentially equal the contribution of 10% of GSCs 336/337 budget to the Reallocation Exercise. Without funding provided by Reallocation, GSCs 336/337 will have no funds to finance grants for new applicants, unless funding levels of renewal grantees is significantly reduced.

Taking into account the major divergence between actual empirical data and the estimate arrived at from NSERC data, we are projecting 35 new grantees per year, including 10 senior grantees.

***Proposal C*: Additional funds for 100 projected new grantees and 40 senior news during the period 2003−2007. This requires $16.5K for each of 100 new applicants and $33K for each of 40 senior news, amounting to a total of $2.97M per year**.

## Part 3: Report on the Canadian Community

This section presents a selected overview of recent Canadian mathematical research in the areas of Fundamental Mathematics, Applied and Industrial Mathematics, and Computational Mathematics. In each case, the presentation will focus on significant researchers by sub−discipline, who were selected with a view to verifying key assertions surrounding our funding proposals. Among our goals are to highlight:

- The very striking interdisciplinary trend in Canadian mathematics. Every area demonstrates major research activity based on interactions with other disciplines.
- The large number of excellent younger researchers in Canadian mathematics. These ~60 younger researchers are identified by the asterisk symbol (*) in this text.
- The presence of a number of active structured groups in Canadian mathematics, focused on training and larger scale research. Many younger researchers are part of these groups.

***Awards***: A Canadian mathematician can be recognized for excellence in a number of ways. Being invited to speak at the International Congress of Mathematicians (ICM) or the International Congress on Industrial and Applied Mathematics (ICIAM), held every 4 years, is a very high honour. Within Canada, a Steacie Fellowship is a notable distinction similar to a US Sloan Fellowship; other important Canadian recognitions are Killam Fellowships and election as a Fellow of the Royal Society of Canada (FRSC). In the past decade, mathematicians have been awarded 4 Steacie Fellowships, 11 Sloan Fellowships, and 8 Killam Fellowships. Recent major recognition includes 4 mathematicians who have received membership in the Order of Canada, and one who has been both elected to the Royal Society and awarded the NSERC Canada Gold Medal for Science & Engineering. The affiliation and recognition of individuals is noted below; the bulleted texts indicate specific, notable examples.

## Fundamental Mathematics

***(a) Number Theory***: Number theory is one of Canada's strongest mathematical areas. It has intimate connections with algebra, analysis and geometry and much of its modern development has arisen out of exploring these connections. The subject has been undergoing a remarkable synthesis whereby analytic and algebraic aspects are now understood to reflect a deeper underlying theory.

The theory of "automorphic representations" and their associated L−functions, the analytic properties of which encode profound arithmetic information, is central to these new developments. The Canadian mathematician **R. Langlands** (IAS Princeton) outlined a visionary program for understanding these representations that has shaped the development of the discipline for the past 20 years. **J. Arthur** (Toronto; FRS, FRSC, Steacie, Killam, Guggenheim, ICM'82, ICM'98, Tory Medal, NSERC Gold Medal) is a world leader in the "Langlands program." His work involves harmonic analysis, Lie theory, automorphic forms, algebraic geometry, and number theory. He has achieved a vast generalization of the Selberg trace formula and, with Clozel, has proved the "cyclic base change," a fundamental step in the effort to establish the Langlands program. The Langlands program also played a crucial role in Wiles' solution of the Fermat problem. **H. Darmon** (McGill; Sloan) and his collaborators have applied these methods to obtain new results on a wide class of "Fermat−like" equations and, more recently, have introduced very novel p−adic methods to prove cases of the "main conjecture" for elliptic curves.

In analytic number theory, **J. Friedlander** (Toronto; FRSC, ICM'94), in a series of papers with Iwaniec and Duke, "broke through" the convexity bounds for L−functions associated to GL(2) modular forms. Major technical novelties they presented have lead to a final resolution of Hilbert's eleventh problem and also to a startlingly low bound on the size of Dirichlet L−functions on the half−line. Friedlander and Iwaniec also achieved a spectacular advance on the asymptotic sieve, allowing them to prove that there are infinitely many primes that are the sum of a square and a $4^{th}$ power. **R. Murty** (Queens; FRSC, Steacie, Killam), well known for his work on L−functions of elliptic curves and Artin's primitive root conjecture, heads a number theory group that includes **E. Kani**, **N. Yui**.and 4 PDFs. Other younger researchers in number theory include **H. Kim*** and **C. Consani*** at Toronto.

- ↔ **H. Darmon** heads a number theory group in Montreal, including **E. Goren*** at McGill, and **C. David**, **H. Kisilevsky**, and **F. Thaine** at Concordia. The group has associated members from Laval, Ottawa (**D. Roy**), Queens and Vermont. Training of HQP is a major focus and 13 Masters, 9 PhDs and 19 PDFs have been trained in the past 4 years.
- ↔ **D. Boyd** (UBC; FRSC, Steacie) and **P. Borwein** (SFU; Chauvenet Prize) lead an emerging Vancouver number theory group of striking potential. Besides **W. Casselman** (UBC; FRSC), the group includes many talented younger researchers such as **M. Bennett***, **G. Martin*,** and **V. Vastal*** at UBC **and S. Choi***, **I. Chen*,** and **P.**

**Lisonek*** at SFU. The group makes extensive and sophisticated use of computation (see section on discrete scientific computing, p. 12).

***(b) Algebra***: In the twentieth century, algebra was dominated by a drive towards abstraction and formalization. A significant reversal of this trend is now occurring: algebraists are typically involved in, and motivated by, problems from other areas of mathematics or theoretical physics. Lie theory pervades many subjects, including analysis, differential geometry, particle physics, string theories, conformal field theories, representation theory, and, increasingly, number theory. Canada is particularly active in the development of infinite dimensional Lie algebras. The research of **R.V. Moody** (Alberta; Order of Canada, FRSC, Wigner medal, Kaplan award), from his early work establishing the foundations of affine Kac−Moody algebras to his current work studying quasi−crystals and aperiodic order, demonstrates this linkage to theoretical physics**. T. Gannon*** (Alberta) filled the gap in the conceptual proof of the "moonshine conjectures" for which Borcherds was awarded a Fields Medal. The discoverer and chief exponent of the deep and fascinating subject of "moonshine" is **J. McKay** (Concordia; FRSC). **S. Berman** (Saskatchewan) has energized a whole community of Lie algebraists whose work links almost all important non−associative algebras. Younger members of this community include **Y. Billig*** (Carleton) and **Y. Gao*** (York). The large interactive algebra group at Alberta also includes **A. Weiss** (FRSC) who, using algebraic K−theory and Iwasawa theoretic methods, has contributed to recent work on the equivariant Stark conjectures.

***(c) Geometry/Topology***: The subjects of geometry and topology permeate almost all branches of mathematics, and have diverse and far−reaching interactions with biology, chemistry, computer graphics, and physics. The strong interaction of theoretical physics with geometry and topology is probably the most exciting recent development−gauge theory has revolutionized low dimensional topology, and there has been explosive growth in symplectic geometry inspired by problems originating in physics.

There has been a vigorous development of these areas in Canada. The work of **F. Lalonde** (Montreal; FRSC, Killam, CRC) has been crucial in the understanding of symplectic invariants. His work with D. McDuff (SUNY Stony Brook; FRS, ICM'98) examines the stability and rigidity of Hamiltonian systems. There is an impressive young group at Toronto in the area of symplectic geometry. **L. Jeffrey*** (Toronto; Sloan, ICM'02), with collaborator F. Kirwan (Oxford; FRS), has given proofs of the famous Witten formulas. She and **E. Meinrenken*** (Toronto; ICM'02) have each made fundamental contributions to the relation between quantization and symplectic reduction. **B**. **Khesin*** (Toronto; Sloan) is a major figure in infinite dimensional symplectic geometry and Lie groups.

**J. Hurtubise** (McGill; AMS Centennial Fellowship) and his collaborators have provided fundamental insights on various moduli spaces of maps, notably proving a long standing gauge theoretic conjecture of M. Atiyah (Fields Medal) and J. Jones. At McMaster, **I. Hambleton** (Britton Professor) leads an active geometry group, including 6 PDFs and the newly hired **H. Boden***, specializing in gauge theory and Riemannian geometry. Toronto also has a high profile group in algebraic geometry, including. **E. Bierstone** (FRSC) and **P. Milman** (FRSC, Killam), who have given a constructive proof for one of the most famous theorems in modern mathematics, Hironoka's desingularization result (for which he was awarded a Fields Medal). **M. Kapronov** (Toronto) is noted for his introduction of deep and fundamental structures into mathematical physics and algebraic geometry. UBC houses a young group in algebraic geometry, comprised of **K. Behrend***, **J. Bryan*** (Sloan) and **Z. Reichstein***, who study algebraic stacks, enumerative geometry, and invariant theory, respectively.

↔ Montreal has a well−established geometry/topology group (CIRGET) organized by **S. Boyer** (UQAM), **N. Kamran** (McGill) and **F. Lalonde**. The 12 members working at the 4 Montreal universities include **J. Hurtubise** and **A. Joyal** (UQAM; FRSC, Killam) and the 3 junior researchers, **V. Apostolov*** and **O**. **Collin*** at UQAM and **D. Wise*** at McGill. Since 1998, this group has trained 26 Masters students, 17 PhDs and 24 PDFs.

***(d) Functional Analysis*:** This is an area of international importance and Canadian researchers have contributed some of the best work in the field. The subject area of Operator Algebras is a blend of algebraic, analytic, and topological techniques and has extensive applications in quantum mechanics and computation, physics, and electrical engineering. **G. Elliott** (Toronto; FRSC, ICM'94, Killam, CRC) created and leads an exciting and ambitious program to classify all C*−algebras through K−theoretic invariants. **K. Davidson** (Waterloo; FRSC, Steacie, Killam) is widely known for his fundamental work in nonselfadjoint operator algebras, and structural theorems for triangular algebras. **D. Handelmann** (Ottawa; FRSC, Steacie, Killam) and **I. Putnam** (Victoria; FRSC) have discovered new and important interactions between C*−algebras and topological dynamics. All of these mathematicians are leaders of highly active analysis groups at their home universities. Among younger researchers, **A. Nica*** (Waterloo) and **R. Speicher*** (Queens), are making major contribution to non−commutative probability and its connection to mathematical physics, standard probability, and combinatorics. Geometric Functional Analysis focuses on the structure of Banach spaces proper, and has been stimulated by several important problems that have been unsolved for 50 years. The work of **N. Tomczak−Jaegermann** (Alberta; FRSC, ICM'98, Killam, CRC), involving sophisticated probabilistic and geometric techniques was crucial to the proof that the only Banach space with large self−symmetry is Hilbert space. Younger researchers in functional analysis also include **M. Khalkali*** (Western) and **V. Runde*** (Alberta).

↔ **G. Elliott** (Toronto) is providing strong leadership to a group focused on his classification program. He has a noted record both for training numerous researchers and for attracting international collaboration. In the past 5 years, using a variety of funding sources, he has trained an average of 4 PhD students and 4 PDFs annually.

↔ **N. Tomczak−Jaegermann** (Alberta) is directing a group that studies high−dimensional convex bodies and their asymptotics, and which involves 3 PhD students and 4 PDFs. Additional funding provided by her CRC position and PIMS has enabled this group to evolve.

***(e) PDE/Mathematical Physics***: The Canadian PDE community includes many strong researchers with well−established ties to other mathematics disciplines, such as dynamical systems, mathematical physics, differential geometry, probability, applied mathematics, and scientific computation. Canada has several researchers of international calibre in aspects of mathematical physics linked to quantum phenomena and spectral analysis. **M. Sigal** (Toronto; FRSC, ICM '90, Killam, Synge Prize) is behind some fundamental work on the problem of quantum asymptotic completeness in many−body problems, and is now a contributor to the stability questions in Ginsburg−Landau theory of superconductors. **J. Feldman** (UBC; FRSC, ICM '90, Synge prize) is a central figure in an international group working on an ambitious program for mathematically rigorous theories of superconductivity and liquid crystals. In a related direction, **V. Ivrii** (Toronto; FRSC, ICM '78, ICM '86), has done fundamental work on the asymptotic distributions of eigenvalues for the Laplace operator. There is a strong group of younger researchers, including **D. Jakobson*** (McGill; Sloan), **V. Jaksic*** (McGill), **I. Laba*** (UBC) and **J. Toth*** (McGill; Sloan), working in spectral analysis with an emphasis on both mathematical physics and number theory.

In the area of dynamical systems, **O. Bogoyavlenskij** (Queens) has discovered a number of striking connections between integrable systems and symplectic geometry. **W. Craig** (McMaster, CRC) works on Hamiltonian PDEs and their applications to the dynamics of surface water waves. Mathematical modelling of signal propagation in optical fibres involves the nonlinear Schrödinger equation, for which **C. Sulem** (Toronto) is an international authority. Geometry and PDE come together in the work of **P. Guan** (McMaster), who has recently completed a proof of conjectures due to L. Nirenberg on the geometry intersections of convex bodies. **J. Chen*** (UBC; Sloan) and collaborators have made fundamental contributions in minimal surface theory, gauge theory and Kahler manifolds, and mirror symmetry. **N. Kamran** (McGill) is currently collaborating with Finster, Smoller, and Yau on important questions in general relativity in the presence of quantum Dirac particles. **R. McCann*** (Toronto; AMS Centennial Fellowship) has been a principal contributor to the recent remarkable progress in understanding the Monge−Kantorovitch transportation problem, posed during Napoleonic times. **N. Ghoussoub** (UBC; FRSC) and **C. Gui** (UBC) have made key contributions to phase transition phenomena by settling a conjecture of Gibbons, then another, related one by DeGiorgi, in dimension 2, and its anti−symmetric case up to dimension 5. **M. Delfour** (FRSC; Prix Urgel−Archambault, Killam) has made major contributions to the control and design of PDEs with numerous applications to technology. His work on shapes and geometry has led to a new intrinsic theory of thin and asymptotic shells with complete relaxation of the classical regularity.

Other impressive younger new researchers include **R. Almgren*** (Toronto), working on pattern formation phenomena; **R. Choksi*** (SFU), specializing in conservation laws and shock waves; **P. Gustafson*** (UBC), studying vortex dynamics; **J. Colliander*** (Toronto), working in harmonic analysis and nonlinear evolution equations; **N. Kevlahan*** (McMaster), specializing in fluid dynamics and scientific computations; **A. Nachman*** (Toronto), dealing with fundamental contributions to inverse problems; **M. Pugh*** (Toronto), a world expert on fluid dynamics; and **A. Tourin*** (Toronto), working with Hamilton−Jacobi equations and their applications.

***(f) Discrete Mathematics*:** Discrete mathematics is essentially the mathematics of computer science and is an area firmly based in applications to engineering and the mathematical sciences. Experts in the subject are found in industry and in many university departments (mathematics, computer science, business, and engineering science) and are funded by many NSERC GSCs. Within Canada, there are large active groups of GSC 336/337 grantees in many basic areas of discrete mathematics, such as algebraic combinatorics, graph theory, finite geometry, graph algorithms, combinatorial designs, coding theory, and, more recently, cryptography. The achievements of the strong cryptography group at Waterloo (notably **R. Mullin** and **S. Vanstone** (FRSC, NSERC Industrial Chair)) are discussed below in the section on Discrete Scientific Computing (p. 12). This group also includes **D. Stinson** (NSERC Industrial Chair), who is funded by a computer science GSC.

Canada has significant strength in algebraic combinatorics, centered at UQAM and Waterloo. **C. Reutenauer** (UQAM; CRC) is working on a wide variety of projects, such as free Lie algebras, combinatorics of words, noncommutative algebras, and descent algebras. **D. Jackson** (Waterloo) is currently studying ramified coverings of surfaces and Hurwitz numbers. **F. Bergeron** (UQAM) is studying families of specialized polynomials: Macdonald, harmonic, quasi−symmetric. **N. Bergeron*** (York; CRC) has a variety of interests, including descent algebras, Schubert polynomials, Hopf algebras, and the geometry of flag manifolds. Canadian mathematicians are making significant contributions to graph theory and combinatorial designs. **B. Alspach** (Regina) is heading a group of graduate students and PDFs that has completely solved the long−standing cycle−decomposition problem. Notable younger researchers in discrete math include **J. Brown*** (Dalhousie), **P. Haxell*** (Waterloo), **A. Menezes*** (Waterloo**), J. Geelan*** (Waterloo), **J. Morris*** (Lethbridge; UFA), **B. Stevens*** (Carleton), **J. Huang*** (Victoria), and **O. Oellermann*** (Winnipeg).

↔ **F. Bergeron** and **C**. **Reutenaue**r are principal figures in a Montreal combinatorics and theoretical computing laboratory (LACIM). The total group is very active in HQP training, with 27 Masters, 13 PhDs, and 6 PDFs in the past 4 years.

***(g) Probability***: Over the past 20 years, new ideas in probability have been driven by interdisciplinary and industrial applications. The subject has been led in many new directions due to its connections with mathematical biology, finance, statistical physics, polymer chemistry, fluid flow models, and telecommunications. Canada has an extremely strong group of researchers in probability who are at the forefront of many of these developments.

**E. Perkins** (UBC; FRSC, ICM'94, Steacie, CRC) and his colleagues have made superprocesses a central subject of international study, in particular by demonstrating that they arise as scaling limits of models used in population genetics, epidemiology, and mathematical ecology. The Canadian probability and statistical physics communities have been strengthened with the recent appointment of **D. Brydges** (UBC; CRC), who has had a profound impact via the introduction of ideas from constructive field theory into probability. One of the most powerful methods in statistical mechanics, introduced by Brydges and Spencer in the mid−1980s, is the lace expansion. **G. Slade** (UBC; FRSC, ICM'94) is internationally renowned for his use of the lace expansion to study polymer models (self−avoiding walk, lattice trees) in chemistry and percolation; his recent work helps establish exciting connections between these models and superprocesses.  An outstanding contribution of probability to fluid mechanics was the derivation by **J. Quastel*** (Toronto; Sloan), together with Yau, of the Navier−Stokes equation as the continuum limit of a many particle stochastic system.

↔ The probability group at UBC is acquiring a new structured presence: it has outstanding core strength with its two CRC holders, Brydges and Perkins, plus Slade, **M. Barlow** (FRSC, ICM'90), and **J. Walsh** (FRSC, ICM'74). More faculty hiring and PIMS support is planned, and CFI funding is being sought to provide appropriate space.

## Applied and Industrial Mathematics

***(a) Biological and Medical Mathematics***:  Just as the 20th century witnessed a revolution in physics, the 21st is destined to be the "Century of Biology." The sequencing of the human genome is expected to lead to major advances in the areas of health, heredity, and the treatment of genetic diseases. The success of the human genome project relied on mathematical and computational ideas, on sophisticated computational algorithms, and on enormous computing power. The major question now is how to extract useful information from these sequences. The identification of the regulatory processes built upon the genome is the next step, defining the emerging area of functional genomics.  Canadian mathematicians involved in this task include **D. Sankoff** (Montreal; CIAR Fellow, Evolutionary Biology Program 1992−2002, FRSC, Killam), **D. Bryant** (McGill), **M. Li** (Waterloo), **P. Kearney** (Waterloo), **V. King** (Victoria), **T. Wareham** (Memorial), **A. Bergeron** (UQAM), **N. El−Mabrouk** (Montreal; CIAR Scholar, Evolutionary Biology Program), and **A. Bonner** (Toronto).

The genome project is part of a global effort that includes research in physiology, cell biology, development, structural chemistry, and genetics to cure or ameliorate disease. Canada has a long established tradition in mathematical physiology and mathematical population dynamics, and these areas continue to make major research contributions. The **Centre for Nonlinear Dynamics in Physiology and Medicine** (**CNDPM**) at McGill is a striking example of such structured interdisciplinary research in Canada. **M.C. Mackey** (FRSC; Joseph Morley Drake Professor) and **L. Glass** (McGill; FRSC, Isadore Rosenfeld Chair in Cardiology) made the conceptual breakthrough in 1977 that popularized the field now known as "Dynamical Diseases." Mathematics members of the CNDPM include **J. Bélair** (Montreal), who uses delay−differential and functional differential equations to model complex biological rhythms (neuromuscular control, cell population, neural networks, and controlled drug delivery), and **S.A. Campbell** (Waterloo), who uses nonlinear dynamical systems and differential delay equations to study regulatory feedback systems in mechanics and physiology. In collaboration with Engineering and Industry (Cardianove), **M. Delfour** (Montreal) is involved in a joint CRM−CERCA NCM_2 project on the design and control of medical devices (e.g. endoprotheses in cardiac surgery). UBC also has a strong tradition in mathematical biology, with prominent pioneers such as **C. Clark** and **D. Ludwig** (management of natural resources, ecology), **R. Miura** (neurophysiology) and younger rising researchers, such as **L. Edelstein−Keshet** (development, pattern formation), **Y−X. Li** (neurophysiology), and **M. Doebeli** (ecology). **R. Miura** (FRSC, Guggenheim), co−founder of the theory of solitons, is a leading figure in mathematical properties of excitable media and has analyzed bursting electrical behavior in pancreatic cells and electrical properties of cortical neurons. In Victoria, mathematical biology is centered on **P. van den Driessche** (population dynamics, epidemic models) and **R. Edwards** (neural networks, neuromotor control). **J. Wu** (York; CRC) and **A. Longtin** (Ottawa) work on modelling in the nervous system, the bifurcation behavior for delay−differential equation models, and especially the role of stochastic processes in signal recognition and transduction. Alberta, under the influence of **H. Freedman,** has also been establishing strong leadership through the recruitment of rising stars such as **M. Lewis** (CRC) in ecology and **G. de Vries** in physiology. Alberta researchers **M. Y. Li***, **J. Muldowney**, **G. de Vries**, and **D. Wiens** work on a MITACS project concerning mathematical modelling in pharmaceutical development.

Canadian leadership has been very visible at the international level through Canadian involvement in the international *Society for Mathematical Biology* (SMB), the most "mathematicized" organization in this area. M. Mackey was the first SMB president, L. Glass and L. Edelstein are past presidents, and M. Lewis is the current president.

***(b) Applied Mathematics and Modelling***: Many problems in the physical sciences and biological sciences can be formulated in terms of partial and ordinary differential equations. The analysis of these equations and their solutions (e.g. regularity, bifurcation and asymptotic behavior, numerical approximations, optimization) can often provide insight into not only mathematical issues but also the physical phenomena under consideration, thereby having a substantial impact on industrial applications. Through the hiring of young talented researchers, there has been a growing trend towards this type of interdisciplinary research in Canada.

In the field of continuum mechanics, **J. Heywood** (UBC), with Rannacher, provided the first full convergence analysis of finite element schemes for the Navier−Stokes equations. These theoretical advances are the basis of an acclaimed CFD solver. Key results of **R. Illner** (Victoria) include a global existence and uniqueness result for the Boltzmann equation. **T.B. Moodie** (Alberta) was the first to introduce strain energy concepts into cardiovascular research. He solved a 50−year old problem by providing a frame invariant form of the constitutive equations of thermo−elasticity. Many talented younger researchers are working in emerging areas of applied mathematics. **M. Ward** (UBC; Steacie, ICIAM ’95) has made important contributions to metastable phenomena in reaction−diffusion equations with applications to materials science. Jointly with condensed matter physicists, **S. Alama** (McMaster) and **L. Bronsard** (McMaster) are working on SO(5) models of superconductivity. Dendritic and crystal growth has been studied by **J.J. Xu** (McGill) with NASA funding. **A. Pierce** (UBC) has made key contributions to the computational analysis of problems in rock and fracture mechanics and has research ties to Schlumberger. **W. Langford** (Guelph) is a leading expert in bifurcation theory, and has studied such diverse topics as the transition to turbulence in the famous Taylor−Couette experiment in fluid mechanics, resonance phenomena for flow−induced vibrations, chaotic mode interactions, and instabilities in biological systems. **A. Lewis** (SFU) has provided, with **J. Borwein** (SFU; Chauvenet Prize), key convergence results for maximum entropy methods.

↔ The newly founded **Applied and Industrial Mathematical Sciences** (AIMS) lab at McMaster, with **W. Craig** as director, is concentrating on the applications of analysis and scientific computing to applied, engineering, and industrial problem−solving. The core members of the group belong to the newly revitalized applied mathematics group at McMaster, which is comprised of **S. Alama**, **L. Bronsard**, and **G. Wolkowicz**, plus 3 recent hirings (**D. Earn***, **N. Kevlahan***, and **D. Pelinovsky* )** and 6 PDFs. Other centres promoting industrial mathematics are the **Applied Mathematics Institute** (AMI) at Alberta and the **Institute of Industrial Mathematical Science** (IIMS) at Manitoba.

***(c) Financial Mathematics***: During the past decade, financial mathematics has become a major area of focus in Canada. **U. Haussmann** (UBC) is a leading figure in optimal control for stochastic differential equations with applications to math finance. The Clark−Haussmann formula in stochastic calculus is one of the fundamental tools used in the pricing of derivatives. Graduate programs have been established at a number of universities and the Fields Institute has made financial mathematics a priority of its outreach program. The **Mathematics Finance Laboratory** at Calgary, **Risklab** at Toronto, and the **Mathematics Finance Laboratory** (PhiMAC) at McMaster are applying the theory of PDEs to the study of market and credit risk. Two MITACS projects, under the direction of **U. Haussmann** (UBC) and **T. Hurd** (McMaster), are investigating financial risk management.

## Computational Mathematics

Computing has now become an integral aspect of science and engineering, playing a role as crucial as theory and experiment have done in the past. This is an area where Canadian mathematics, both pure and applied, has displayed considerable innovation. Leadership has been demonstrated in numerical modelling, computational number theory, cryptography and symbolic analysis. Number theory with its major ties to algorithms, cryptography, error correction, and computing, is a major theme in much of the work which we detail.

### *(a) Numerical Modelling*

**M. Fortin** (Laval; FRSC) is a leading figure in mixed finite element methods and an original contributor to computational fluid and solid Mechanics. He has established a long standing and effective cooperation with Engineering and Industry (e.g., Bombardier, ADS Composites, Prevost Cars) and with medicine through interdisciplinary projects like SKALPEL−ICT (simulation kernel applied to the planning and evaluation of image−guided cryo−therapy), and tackled large scale industrial problems (PIGE−ISP, Pratt & Whitney, ALCAN). **A. Fortin** (Laval) is involved in the forming of polymeric materials and the development of iterative solvers for fluid flow problems (Peugeot and Elf). He is currently working on mixed methods for large deformation of hyperelastic materials with Michelin.

↔ **Giref** (**Groupe Interdisciplinaire de Recherche en Elements Finis**) at Laval University under the direction of A. Fortin, promotes and develops mathematical techniques for engineering sciences, particularly through joint development of software. Industrial contracts include Bombardier, ALCAN, and Pratt & Whitney Canada. Giref has a budget of $230K, and organizes the very successful annual "*Journée des éléments finis*."

The **Mathematical Modelling and Scientific Computation** group is part of the MITACS NCE and consists of **R. Choksi*** (SFU), **H. Huang** (York), **M.C. Kropinski** (SFU), **R. Miura** (formerly UBC), **A. Peirce** (UBC), **K. Promislow** (SFU), **R. Russell** (SFU), **S. Ruuth** (SFU), **B. Seymour** (SFU), **M. Ward** (UBC), **B. Wetton** (UBC) and **R. Westbrook** (Calgary). They have an on−going project with Ballard Power Systems, with whom they are modelling multi−dimensional heat and mass transfer with phase change in fuel cell electrodes. The group of **M. Best**, **L. Tuncel**, and **H. Wolkowicz** in Waterloo is specialized in continuous optimization, semidefinite relaxations for computationally hard problems, quadratic programming, and optimization, and interior−point methods. Other labs for computational mathematics are the **Numerical Analysis Research Laboratory** at Ottawa and the **Combinatorial Computing Center** at Memorial.

### *(b) Discrete Scientific Computing*

**(i) Symbolic Computation**: In Canada, symbolic computation has been considerably advanced through the work of **J. Borwein** (SFU; FRSC, Chauvenet Prize), most notably through his role as director of the **Centre for Experimental and Constructive Mathematics** (CECM) at SFU.

↔ CECM researchers explore and promote the interplay of conventional mathematics and modern computation. The CECM pioneered this area in Canadian mathematics and remains a driving force. It supports research projects connected with symbolic computation, numerical computation, complexity, collaborative network technology, digital information and visualization. CECM projects involve many industrial partners, including MathResources, Waterloo Maple, SmartTech, SynchroPoint, and Packeteer.

A MITACS project on Symbolic Analysis headed by **P. Borwein** (SFU; Chauvenet Prize) is an outgrowth of CECM innovations. The project, with Maple as a corporate partner, is exploring the use of symbolic analysis in mathematical analysis and includes the active participation of grantees from a variety of disciplines, notably **S. Watt**, funded by Computing Science, and **R**. **Corless**, funded by Mechanical Engineering, both at Western Ontario.

**(ii) Computational Number Theory**: The growing reliance of theory upon computation is a striking trend in current number theory. There has been ongoing development of algorithms and conjectures based on computation. **P. Borwein** (SFU; Chauvenet Prize), in part with **J. Borwein** (SFU; Chauvenet Prize), has made striking advances in algorithms for number theory, notably with an amazing and efficient way of computing specific digits of polylogarithms (including pi). The study of L−functions is one area that has greatly profited from the use of computational techniques. **D. Boyd** (UBC) has provided computational insights into the special values of L−functions and their relation to Mahler measures, plus explored a whole range of other computational issues: sphere packings, Pisot and Salem numbers, and invariants of hyperbolic manifolds. **M. Bennett*** (UBC) and **G. Martin*** (UBC) have developed special purpose algorithms in their respective studies of diophantine equations and zeros of L−functions.

**(iii) Cryptography**: Cryptography has a major presence in Canada. Number theory and algebraic geometry have formed the basis for the very successful public cryptosystem developed by the Waterloo mathematicians **R**. **Mullin** and **S. Vanstone**. Their company, ***CertiCOM***, even after the market turndown, employs approximately 200 people, including 20 mathematical and engineering PhDs; Revenue Canada, IBM, Motorola, and various banks are among its customers. The number theorist, **K. Murty** (Toronto; FRSC, Steacie), has commercialized an encryption system based on algebraic varieties. His company, ***Karthika***, established in 1999, now employs 15 people. MITACS projects, with Murty and **A. Menezes** (Waterloo) as leaders, are also dedicated to cryptography. **H. Williams** (Calgary; Killam) has made a number of advances in primality testing and public key cryptography, and is head of the **iCORE Centre for Cryptography** at Calgary.

↔ **CACR (Center for Applied Cryptographic Research)** at Waterloo, founded in 1998, has quickly grown to become one of the largest cryptographic research centres in the world. Between 1998 and 2001, CACR received over $1M in research funding from private industry (including Certicom, MasterCard, and Pitney Bowes) which was matched by NSERC, MITACS, ORDCF and CITO. In 4 years, CACR has organized 7 international research conferences and 9 one−day information security workshops for its industrial partners.

Other labs devoted to computer mathematics are the **Laborative de Combinatoire et d'Informatique Mathematique** (**LACIM**) at UQAM and the **Applied Computer Lab** at Saint Francis Xavier.

## Part 4: Mathematical Institutes

The Canadian Institutes for the Mathematical Sciences are pivotal forces that shape and promote the new reality for Canadian mathematics (as described in Part 1). The 3 Institutes have provided innovative leadership at the international level by organizing an amazing variety of scientific activities with an emphasis on emerging areas, areas at the interface of different branches of mathematics, and areas that cross traditional discipline boundaries. They have been particularly effective in the promotion of interdisciplinary research, and have systematically built partnerships among the mathematical sciences, other disciplines, and the business/industrial sector. As well, the Institutes represent the principal mechanism for outreach by the mathematics community to both industrial and educational sectors. Finally, there has been a significant commitment to comprehensive training of HQP, both at graduate and postdoctoral levels, with a particular focus on the types of significant and innovative research described above. The cumulative effect of the Institutes on Canadian mathematical life and on the role of mathematics in Canadian science has been profound.

Canada’s Mathematical Institutes have evolved a unique capacity to function simultaneously as both a national network and as 3 distinct regional presences. The Institutes provide significant benefits to Canadian mathematics at both levels, and major initiatives have been created both by a single Institute (NCM_2, BIRS) and by the Institutes as a whole (MITACS). The Institutes’ regional base has enabled each to leverage resources and to multiply opportunities at a remarkable rate. Currently, federal funding for the 3 Institutes is being leveraged by almost $4M per year from provincial governments and universities and by almost $3M per year from the private sector. A regional presence also ensures maximal leveraging of human resources. Research institutes have a major impact−and a major reliance−upon scientific activity in their home region. Furthermore, regional presences have been crucial to effective outreach, which is primarily focused upon local business and industry.  At the national level, there is significant collaboration between the Institutes with the objective of creating research opportunities in all of Canada’s regions. MITACS (to be discussed in Part 5) is their largest and most successful joint venture. The Institutes also fund and coordinate their National Program Committee, which provides support for the Atlantic Association for Research in the Mathematical Sciences (AARMS) and for 3 major professional Canadian mathematical societies. One recent initiative of the 3 Institutes has been the partnering with several Atlantic Canadian universities and AARMS to develop mathematics in Eastern Canada. The goal is to develop a series of graduate activities and workshops to foster mathematics in a region with many smaller universities.

The Institutes not only collaborate, but also compete in appropriate spheres. We view such competition as healthy and proper, as it is part of a striving for excellence and is clearly contributing to the ongoing innovation displayed by the Institutes and to their considerable impact on the Canadian and international mathematical scene. *In summary*, the Institutes are thriving, and are much appreciated by Canadian mathematicians. The 3 Institutes play a central and essential role in Canadian mathematics and their impact will continue to grow in the coming period. The main contributions of each Institute since the last Reallocation Exercise will be outlined in the Appendix at the end of this document.

## Part 5: New Initiatives in Infrastructure

Interdisciplinary activity is now a major force shaping both Canadian mathematical research and graduate training. Most of this interdisciplinary thrust is attributable to a new level of infrastructure created over the past decade. The mathematical institutes, both in their own right and through a number of major initiatives, have displayed major leadership in this area. This section begins with 3 of their most comprehensive interdisciplinary initiatives.

**MITACS:** The highly successful national network **Mathematics of Information Technology and Complex Systems** (MITACS) is a joint initiative of the 3 Institutes. MITACS has fostered a broad range of collaborative partnerships involving the mathematical sciences and industry/business. It currently funds 25 projects, with more than 75 corporate sponsors. Over 40 grantees from GSC 336/337 have participated in 15 different MITACS projects that have involved 150 scientists and 50 corporations or companies. GSC 336/337 has provided project leaders or co−leaders for 9 of these projects, which are investigating such diverse phenomena as seismic imaging, symbolic analysis, pattern storage retrieval and recognition, cryptography, material science, signal processing, and financial risk management. The projects currently receive $825K from MITACS and $300K from other sources.

**BIRS:** The **Banff International Research Station** (BIRS) is a new research facility located in Alberta's Banff Centre. BIRS is based on the highly successful Oberwolfach/Luminy model (comparable to the Boston−based Gordon Research Conferences). Beginning in 2003, BIRS will host 40 international 5−day research−intensive workshops each year, covering all pure and applied mathematics, statistics, mathematical biology, mathematical physics, theoretical computer science, industrial and financial mathematics, and promote training of young, talented researchers. BIRS is a joint Canada−US collaborative venture of PIMS and the MSRI (Berkeley). In addition to PIMS support, 4 years funding has been secured from the NSF, NSERC (MFA), and the Alberta Science Research Authority. The Call for Proposals for 2003 had an enthusiastic response, with 108 quality proposals for workshops, each having a Canadian and an American co−organizer. The initiative is the first joint NSERC/NSF venture of this scale.

**NCM_2:** The **Network for Computing and Mathematical Modelling** (NCM_2) is an alliance of 8 research and transfer centres in the Montreal area; CRM is the lead partner. NCM_2's 5−year, $3M NSERC research partnerships program grant, with an equivalent contribution in industrial funds, currently supports 20 projects in areas such as optimization, finance, risk management, imaging, and data−mining. NCM_2 has established large−scale laboratories with some major partners. These include a $12M laboratory on electronic commerce and multi−media research, launched jointly with Bell in 1998, and, most recently, the Laboratoire Universitaire sur le Temps Extrême (LUTE), which has annual contributions of $300K cash, 5 full time researchers, and up to $1M in computing time from Environment Canada. In addition, there are two large (>$1M) Valorisation Recherche Québec projects, in e−commerce and risk management

**New Labs and Centres:** Another confirmation of the trend towards interaction is the evolution over the past decade of a striking number of new labs or centres dedicated to interdisciplinary activity and industrial mathematics. These labs and centres have base funding, are typically university−based, and reflect the emergence of new research and training priorities at the university level. Ten years ago there were only a few such labs or centres; today, there are at least 20. All of these new labs and centres involve extensive participation by researchers funded by GSC 336/337. A number of these labs and centres were discussed in Part 3. There are also at least 20 additional, less structured, groups, distributed over 10 universities and mathematics departments. These groups pursue interdisciplinary research on topics such as dynamical systems, mathematical modelling, mathematical biology, mathematical physics, numerical analysis, scientific computing, and telecommunications algorithms. These groups have had a number of CRC chairs assigned to them. The primary need of these groups is significant long term funding, particularly to expand their training capabilities.

## Part 6: Highly Qualified Personnel

Mathematical training of HQP in Canada shows two noticeable trends: interdisciplinary training and training of PDFs.

### Interdisciplinary Training

The most significant development over the past 4 years has been the growth in infrastructure for interdisciplinary training. Interdisciplinary training in mathematics is undergoing rapid evolution at many Canadian universities. Successful Masters programs in Financial Mathematics have been initiated at Alberta, Calgary, Toronto, UBC, and Waterloo; McMaster plans one. Masters programs in Industrial Mathematics have been developed at Ottawa/Carleton (high technology), Manitoba, Memorial (computational science), Montreal, and Toronto; McMaster plans one also.

A great deal of interdisciplinary graduate training in mathematics departments is now carried on by utilizing the resources of the various labs and centres described in previous sections. In particular, these labs and centres have enabled certain departments to enrich considerably their interdisciplinary offerings for graduate students. For example, the **IAM** at UBC, with 69 faculty members from 13 different departments currently supervises 30 graduate students from the Department of Mathematics. Since 1997, **Giref** at Laval has graduated 49 Masters, 33 PhDs, and 22 PDFs in engineering and mathematics; it is currently training 35 Masters, 47 PhDs, and 4 PDFs. The establishment of **CACR** at Waterloo has given a new focus in Canada to training in cryptography. Since it was founded in 1998, the CACR at Waterloo has trained 20 PDFs and over 30 graduate students. The SFU **CECM** has trained more than 40 PDFs and Research Associates since its inception in 1993.

A recent survey gave the following employment patterns for graduates between 1997 and 2001:

- Of 219 PhDs, 139 accepted academic positions (40 PDFs; 68 tenure−track, 23 limited term, and 8 college appointments), 56 entered the private/public sector (15 in finance, 36 in high tech industry, and 5 in government), 15 retrained, and 9 are unknown.
- Of 333 Masters students, 125 pursued a PhD in mathematics, 113 entered the private/public sector (29 in finance, 64 in high tech industry, and 16 in government), 26 retrained, and 29 are unknown.

Future expansion in Canadian mathematical graduate training will support a growing number of mathematical students who are being employed in the financial/industrial world. The 1995 SIAM "*Report on Mathematics in Industry*" cites the fact that industrial managers repeatedly commented on the importance of mathematicians for their analytical thinking skills and "*expertise for formulating and solving problems in different contexts*."

Interdisciplinary research and training has broad needs and, as a result, is more expensive than are other areas of mathematical training. One obvious requirement is large−scale computing facilities. As well, there is a need to provide students with continuous exposure to innovative research that arises from other disciplines and the industrial sector. This requires face−to−face contact with other scientists and representatives from industry. The major Institutes and MITACS, with their extensive contacts and experience, have played a major role in providing forums for such interactions.

### Postdoctoral Fellows

Training of PDFs is a major success story in Canadian mathematics. In the past decade, there has been a striking rise in the numbers of PDFs trained. Data from the 1996 Review of Canadian Mathematics shows 370 PDFs trained in Canadian mathematics departments during the 5−year period 1991−1996, and a recent survey shows that GSC 336/337 grantees (representing 80% of mathematicians with grants) alone trained 560 PDFs during the 5−year period 1997−2002. This very impressive 50% increase is clearly an index of increased mathematical activity and of a strengthened discipline.

The increase in PDF training is also linked to the importance of new funding sources. In GSC 336/337, even the larger grants are not sufficient to support a PDF fully, so that additional funding has been required to complement research grants. In particular, the 3 Institutes and MITACS are providing such partial funding: each year, on average, the Institutes provide $860K to support 60 PDFs and MITACS provides $1.4M to fund 100 PDFs from a variety of disciplines. Moreover, much of the extra funding provided to highly active researchers in the last Reallocation Exercise was leveraged into funding packages for the support of PDFs.

As these statistics illustrate, recruitment of PDFs is a priority in Canadian mathematics. PDFs play a crucial role in mathematical research; in particular, a critical mass of PDFs is a crucial factor in the creation of highly effective research teams, as indicated by the examples given in Part 3. As potential new faculty, the large international pool of talented PDFs in Canada also provides an important resource for Canadian universities. For example, 4 of the faculty in the Vancouver number theory group (see Part 3) were hired from PDF positions in Canada.

## Part 7: Further Considerations on Funding Proposals

This section provides further comment on funding proposals A, B, and C presented in Part 2.

### New Applicants

In this section, we explain how NSERC data supports a projection of 25 new grantees per year in GSCs 336/337 during the period 2003−2007. *Before providing this explanation, we repeat the observation, made in Part 2 (p. 4), that this prediction is at variance with empirical data, which suggests a pattern of at least **35** new grantees each year.* NSERC data predicts that 2000 researchers in the Natural Sciences will be hired by Canadian universities over the next 5 years (BP, New Applicant Survey), and that 1 out of 7 of these researchers will be hired by mathematics and statistics departments (BP, Figure 7). Current patterns lead one to expect that 2 out of 3 of those hired will be mathematicians (as opposed to statisticians) and that 80% of these will apply to GSCs 336/337. Finally, a success rate of 85% among new applicants in GSCs 336/337 leads to the figure of at least 25 new grantees per year.

The consequences of no increase in funding of our proposals for GSCs 336/337 will be severely detrimental to Canadian Mathematics. We first want to emphasize that the goals of these funding proposals cannot be realized unless funding is provided by the Reallocation Exercise. There are no other funds available in the GSC budget unless continuing grantees in GSCs 336/337 are to be deprived of needed funds.

### Attrition

NSERC data indicates that, during the period 2003−2007, the funds made available by projected attrition will almost exactly balance the amount that Mathematics GSCs will contribute to the Reallocation Exercise. Thus, any money made available by attrition over the next 4 years is effectively gone and will be returned only through funding of our proposals.

If the pattern of attrition for the period 1997−2001 (BP, Table 6) is superimposed onto the current grantee population (BP, Table 3), one predicts that 94 grantees will not reapply during the period 2003−2007. According to Table 6 of BP, those who withdrew in the years 1997−2001 had an average grant of $10,340. Thus the departure of 94 grantees in the period 2003−2007 is predicted to provide GSC 336/337 with roughly $970,000.

Notably, there will be little attrition associated with large grants during the period 2003−2007. One indication of the longevity of the top researchers (the 59 grantees in Mathematics with grants of at least $30K) is that only one has taken early retirement and only two will reach retirement age before 2007. The funding needs of these top researchers will certainly not diminish. Indeed, the argument of Proposal A is that many of these researchers are already under−funded.

### Consequence of No Funding

If Proposals A and B are not funded, then these initiatives would be deferred, many opportunities for innovation in Canadian mathematics would be lost, and resources for leaders and emerging leaders would remain inadequate. If Proposal C is not funded or is funded at a low level, then GSCs 336/337 will have an overwhelming problem in dealing with new applicants, many outstanding researchers will be under−funded, and the momentum achieved to date will be lost.

**Additional Note**

Our funding proposals reflect priorities for Canadian mathematics over the next 4 years. We have made an effort to balance innovation and continuity while pursuing these priorities. We believe that a balanced funding of all our proposals would maintain maximal flexibility for GSCs 336/337 and best preserve both the health of our discipline and its continued growth and enhancement.

## Appendix: Details on the 3 Mathematical Institutes

### Centre de Recherches Mathématiques (CRM)

Founded in 1969, the CRM has been a leader in mathematical innovation in Canada for the last 20 years. CRM developed the national institute model for research in the mathematical sciences, with a solid core program in mathematics, a strong international profile, and a wide network of collaborators in other disciplines and in industry. It currently receives $874K in NSERC funding and has a total budget of $3M per year. It is based at the University of Montreal, with principal partners the Montreal universities, and affiliations with nearby universities, such as Ottawa, Queens, and Laval.

At the heart of the CRM's activities lies a **thematic program**, chosen each year for its quality, timeliness, and pertinence to the development of mathematics in Canada. We give highlights from two examples. Last year's program *on Mathematical Methods in Biology and Medicin*e focused both on the intricate and sophisticated modelling issues for various physiological processes, and on the difficult mathematical and statistical problems of inference posed by genomics. There were 11 workshops, a summer school, 2 large international meetings, 6 courses and seminar series, and Aisenstadt lectures by two of the most eminent specialists in the area, A. Winfree and M. Waterman. This program illustrates the interdisciplinary flavour of many of the CRM's programs, as it brought in >1500 participants from mathematics and other areas, such as physiology, neurology, genetics, physics, and computer science. The current program lies in the core subject of *Groups and Geometry*. A 3−week program on low−dimensional topology in the summer attracted >100 participants for each week, including many students. A similar program, on the Geometric Langlands program, has attracted many of the world leaders in the area (including R. Langlands, who will deliver 8 hours of lectures on his latest ideas in the subject).

The CRM's **general program** covers a wide range of short programs and workshops (some off−site), as well as the joint CRM−FI−PIMS National program, prizes, a colloquium, and a joint PDF program with the local community. The CRM been heavily involved in structuring this community, in particular by the creation of sub−disciplinary laboratories in the Montreal area, which assist the integration of the CRM's international breadth with the activities of Canadian researchers. Last year, the CRM hosted over 100 research visitors, outside of the 41 international conferences, workshops, schools, or short courses of its various programs.

In its interdisciplinary and industrial programs, the CRM is the lead institution for the Montreal−based **Network for Computing and Mathematical Modelling** (NCM_2), which has had some remarkable successes in the construction of large−scale partnerships, detailed below. The NCM_2 will also be organizing two joint workshops and a PDF exchange this year with Minnesota's Institute for Math − and its Applications (IMA), in addition to its usual lectures and small workshops. It has also developed ties with NRC's Industrial Research Assistance Program. Another success for the CRM has been the constitution of a research network in brain imaging in Quebec. A CRM−piloted application for funding personnel is pending; it will complement large CFI initiatives for imaging equipment in which the CRM is a partner; one of these has been awarded $23M. The imaging program led to a CRM spin−off, *Zeugma Technologies*, which was recently awarded $20K first prize in the Odyssée Entrepreneurship competition.

The CRM places great emphasis on training; part of this is through the integration of short courses into most of its scientific activities, and part through specialized activities, such as summer school (1 per year). The other important impact is through sponsoring PDFs: over the last 3 years, it has co−financed an average of 15 PDFs per year, and has been host to 7 more externally funded PDFs per year.

The CRM fully endorses the vision for the discipline presented in these pages, and indeed has anticipated certain of its aspects. The CRM emphasizes both training and disciplinary outreach in its programs, and it has been enthusiastically integrating new researchers into its programs and helping them in a rapid development of their careers.

## The Fields Institute

The Fields Institute has been remarkably effective in developing links among mathematicians, other mathematical scientists, and the user community, in areas from biology to physics to finance. It has an extremely broad scope of activity, not just in its support of mathematical research through its scientific programs, but also in math education and in commercial and industrial development of mathematical research. The Institute is there to help the mathematical community cross discipline boundaries and to reach out to the rest of the world. Its activities mesh extremely well with the current goals of the Mathematics GSCs.

The central scientific activities of the Fields Institute are based on **thematic semesters and years**. These programs bring together top researchers from around the world to participate for an extended period in a supportive research environment. Programs focus on areas with exciting current activity, and seek to juxtapose researchers with related interests across the spectrum from pure to the applied. A good example was the recent year−long program in *Probability and Applications* (to physics, communications, finance, and biology) which brought together mathematicians and statisticians with researchers applying these methods in diverse areas. The 1999-2000 program in *Graph Theory and Combinatorial Optimization* attracted mathematicians and computer scientists from both academia and industry. These programs attract the best and brightest of the Canadian mathematical community as both organizers and participants. It is usual to build a program around Canadian strengths, and many top senior and junior mathematicians have been involved. Long−term programs give a significant boost to research efforts, and foster training of new researchers through advanced graduate courses, seminars, and PDF support. Many individuals identified as emerging leaders in this report, and in the report 4 years ago, have been involved in these programs.

Today's mathematicians collaborate extensively. They require the opportunity to get together and exchange ideas. The Fields Institute can provide the infrastructure, logistical support, and supportive environment that is ideal for nurturing this kind of research. Fields is also in a position to react quickly to the latest developments and to mount activity to capitalize on it through workshops, seminars, short courses, and lecture series. Summer schools are a new avenue that is highly effective for trainees to learn about cutting edge research, and plans are in place to expand this program. A very successful summer school in *Quantum Information Processing* was held last year, with a highlight Distinguished Lecture Series by P. Shor (Bell Labs), winner of the 1999 Godel prize. An upcoming summer school in *Automorphic Forms and the Trace Formula* will be taught by J. Arthur, and be funded in part by the Clay Institute.

The Fields Institute has a tremendous breadth of activity that goes well beyond the scientific programs funded by its NSERC grant. The 3 Institutes founded the National Centre of Excellence, MITACS, which now funds over 20 projects in the mathematical sciences across Canada. Many of the more applied mathematicians involved in groups have taken advantage of this remarkable new program; it has enabled groups to develop at many centres. In addition, Fields runs an incubator program for commercial projects. Two successful start−ups involved K. Murty and L. Seco, both leading Canadian mathematicians. The latter has led to a joint major ORDCF proposal in Energy Finance with 4 Ontario universities.

On the other side, Fields is one of the few mathematics research institutes in the world that is also concerned with mathematics education below the university level. Its Math. Ed. Forum has been the basis of a number of projects including curriculum development in Ontario. A national effort focused on the use of technology in the teaching of mathematics is currently applying to SSHRC's Initiative on the New Economy program for funding.

The Fields Institute is working to develop programs and interaction in a timely way over the whole range of mathematical sciences. The raw material for this interaction is the very strong and vibrant Canadian mathematical community. Working together, we can do a lot of exciting science, strengthen connections between branches of pure and applied mathematics, and reach out to the broader scientific community.

### The Pacific Institute for the Mathematical Sciences (PIMS)

PIMS was created in 1996 and has evolved into a unique bi−national scientific partnership, involving all of the major universities in Alberta, BC, and Washington. This appendix describes PIMS pioneering efforts in creating domestic and international partnerships. The PIMS independent submission deals with the Institute's distinctive scientific programs and its industrial and educational initiatives. In the last 3 years, PIMS undertook a series of bold national and international actions−MITACS, the Banff Research Station, the Pacific Northwest Partnership, and the Pacific Rim Initiative−that have markedly raised the visibility of our community. These initiatives have also multiplied the opportunities for the mathematical science community in Canada and the world and attracted substantial funding from industrial, provincial, federal, and foreign sources in support of Canadian−led research.

By sharing resources among its participant universities**,** maintaining a flexible program structure, and empowering its membership, PIMS has achieved a much higher level of activity, innovation, and opportunity than would be possible otherwise. **NSERC**'s support for PIMS is doubly matched by its **partner universities** and by the **governments** of **Alberta and BC**. PIMS receives (free of overhead) substantial research infrastructure support from its participant universities, including two fully−equipped facilities (each 5,000 sq.ft) at UBC and SFU. Therefore, PIMS is in a strong position to concentrate its resources on research and training. The investment by PIMS in 30 PDFs is matched equally by its industrial partners and affiliated departments. Students from across Canada have access to at least 6 annual PIMS summer schools and training camps in industrial and emerging areas of the mathematical sciences.

With its **US partners**, PIMS developed the **Banff International Research Station**. In addition to the support of PIMS and of the Berkeley−based MSRI, this unique Canada−US joint venture has received funding from NSF, the Alberta Science Research Authority and from NSERC's MFA program. This new facility will greatly multiply the opportunities, as more than 1700 Canadian and international scientists from across the spectrum of mathematical disciplines are expected to participate in BIRS activities every year**.** The initial ventures of the **Pacific Northwest Partnership** are the 12 PIMS− and NSF− sponsored Pacific Northwest (PNW) seminars that help to sustain the collaborative effort throughout Western Canada and the US. Beginning in 2003, the PNW segment of the PIMS industrial program will be developed in collaboration with the Minnesota−based IMA. PIMS is also looking forward to more partnerships with NSF on a number of Pan−American Advanced Studies Institutes.

Together with its **Pacific Rim partners,** PIMS and its sister institutes in China, Taiwan and Japan, are developing **the Pacific Rim Initiative.** Jointly, they organize major Pacific Rim events (Hong Kong '98, Taipei '01, Vancouver '04) promoting scientific links throughout the region**.** The annual PIMS program on *Frontiers in Mathematical Physics* is a joint initiative with the Perimeter Institute (Waterloo), and the Asia Pacific Center for Theoretical Physics (Korea).

The PIMS industrial problem−solving workshops and its industrial collaborative program continue to be key building blocks for the MITACS network. In the last 2 years, 53 PIMS **industrial partners** contributed to the institute's collaborative research program, including 11 out of the 23 MITACS projects**.** In partnership with the Institute for Computing, Information, and Cognitive Systems and the New Media Innovation Centre (*NewMIC*), PIMS is developing a 3−year interdisciplinary program on *Mathematics and Multimedia* to bring mathematicians and statisticians together with their colleagues in computer science and engineering.

With its **participating K−12 schools**, PIMS researchers annually organize dozens of *"math evenings''* an*d "math fairs"* across Western Canada and Washington State. PIMS publishes and distributes "*Pi in the Sky*," a magazine for math educators and students. The *Women and Mathematics* poster campaign followed the highly acclaimed Year 2000 *Mathematics is Everywhere* posters on buses and in classrooms. PIMS is also a pioneer in web−casting major events to the world scientific community; lectures by PIMS distinguished scientists (e.g., A. Huxley, H.S.M. Coxeter, and others) are made available over the internet using streaming video.