WESTERN CANADA LINEAR ALGEBRA MEETING
Program - Abstracts - Participants

University of Manitoba

May 30-31, 2008

Organizing Committee

Shaun Fallat, Hadi Kharaghani, Steve Kirkland, Peter Lancaster,
Michael Tsatsomeros, Pauline van den Driessche

Local Organizer:
Robert Craigen

Funding

WCLAM 2008 is organized with the support of the
Pacific Institute for the Mathematical Sciences
and the
University of Manitoba

Invited Speakers

Prof. Michael Gekhtman, University of Notre Dame
Prof. Olga Holtz, Technische Universitaet Berlin
Prof. David Watkins, Washington State University

Location

E2 330 and E2 350 of the Engineering Building, University of Manitoba

1 Meeting Program (E2 350, Engineering Bldg.)

Friday, May 30, 2008

08:00-08:45	**Registration**
08:45-09:00	**Welcome & Information**
	Chair: Peter Lancaster
09:00-09:50	**David Watkins (invited)**, *Understanding the QR Algorithm, Part IX*
10:00-10:25	**Amir Amiraslani**, *Strong Linearization of Matrix Polynomials in Various Bases*
10:25-10:45	**Break** *(adjacent hallway)*
	Chair: Pauline van den Driessche
10:45-11:10	**Rajesh Pereira**, *Some Results on spectrally Arbitrary Sign Patterns*
11:15-11:40	**Minerva Catral**, *Group Inverses of Matrices with Path Graphs*
11:45-12:10	**Frank Hall**, *Sign Patterns That Require Almost Unique Rank*
12:10-13:45	**Lunch**
	Chair: Steve Kirkland
13:45-14:10	**Murray Bremner**, *Lattice Basis Reduction and Polynomial Identities*
14:15-14:40	**Pauline van den Driessche**, *Bounds for the Perron root using max eigenvalues*
14:45-15:10	**Amy Yielding**, *Spectrally Arbitrary Zero-Nonzero Patterns*
15:10-15:30	**Break** *(adjacent hallway)*
	Chair: Robert Craigen
15:30-15:55	**Dale Olesky**, *Sufficient Conditions for Permutation Equivalence to a WHS-matrix*
16:00-16:25	**Elizabeth Bodine**, *Spectrally Arbitrary Zero-Nonzero Patterns over Finite Fields*
16:30-16:55	**Adam Rogers**, *Sums of Kronecker Products for Compound Magic Squares - Eigenproperties*
18:00-	**Informal Dinner**

Saturday, May 31, 2008

Chair: Shaun Fallat

08:30-08:55	**Robert Craigen**, *Circulant Partial Hadamard Matrices*
09:00-09:50	**Michael Gekhtman (invited)**, *Biorthogonal Cauchy Polynomials, Total Positivity and Matrix Models*
10:00-10:25	**Sandra Fital**, *On the Fixed-point Iteration for the Matrix Equations* $X \pm A^* X^{-1} A = I$.
10:25-10:45	**Break and meeting photo** *(adjacent hallway)*

Chair: Hadi Kharaghani

10:45-11:10	**Juan Carlos Zuniga Anaya**, *Discrete time polynomial* J*-spectral factorization*
11:15-11:40	**Mahmud Akelbek**, *Primitive Digraphs with the Largest Scrambling Index*
11:45-12:10	**Sasmita Barik**, *On Graph Products and the Resulting Laplacian Spectra*
12:10-13:45	**Lunch**

Chair: Michael Tsatsomeros

13:45-14:10	**Pappur Shivakumar**, *Shape of a Drum, A Constructive Approach*
14:15-14:40	**Mahdad Khatirinejad**, *Unbiased Bases Arising from Weyl-Heisenberg Orbits*
14:45-15:10	**George Styan**, *Some Comments on Graeco-Latin Squares and on Magic Squares, Illustrated with Playing Cards and Postage Stamps*
15:10-15:30	**Break** *(adjacent hallway)*

Chair: Steve Kirkland

15:30-15:55	**Peter Loly**, *Two Small Theorems for Square Matrices Rotated a Quarter Turn*
16:00-16:50	**Olga Holtz (invited)**, *Matrix Methods in Stability Theory*
16:50-	**Closing remarks**

Posters

(E2 330 of the Engineering Building)

Michael Cavers, *On Reducible Matrix Patterns*

Ryan Tifenbach, *The Dual of a Graph*

2 Abstracts for talks (alphabetical by speaker)

Primitive Digraphs with the Largest Scrambling Index
Mahmud Akelbek

The scrambling index of a primitive digraph D is the smallest positive integer k such that for every pair of vertices u and v, there is a vertex w such that we can get to w from u and v in D by directed walks of length k; it is denoted by $k(D)$.

There are numerous results giving the upper bounds on the second largest modulus of eigenvalues of primitive stochastic matrices. By using Seneta's definition of coefficients of ergodicity, we have provided an attainable upper bound on the second largest modulus of eigenvalues of a primitive matrix that makes use of the so-called scrambling index and we gave the upper bound on $k(D)$ in terms of the order and the girth of a primitive digraph D. In this talk, we characterize all the primitive digraphs such that the scrambling index is equal to the upper bound.

Strong Linearization of Matrix Polynomials in Various Bases
Amir Amiraslani

In this talk I will consider matrix polynomials represented in various bases such as degree-graded bases (e.g.monomial and orthogonal bases), the Bernstein basis, and the Lagrange basis. I will present the companion matrix pencils resulting from the linearization process in each case. Using the LU factors of these companion matrix pencils I will show that these linearizations are "strong", i.e. that they preserve all of the eigenstructure of the original poynomial.

This talk is mainly based on a joint work with Peter Lancaster and Robert Corless to appear in IMA Journal of Numerical Analysis.

Discrete time polynomial J-spectral factorization
Juan Carlos Zuniga Anaya

In this talk we extend the results of our previous work [1] to the rational matrix case. Then we apply this new algorithm to the J-spectral factorization of matrix polynomials in the complex indeterminate z, where the variable z represents the variable of the Z-transform of a discrete time linear (differential) system, i.e. we can see z as the shift operator (the discrete time version of the differential operator d/dt). Our algorithm is based on a symmetric factor extraction procedure (which can be implemented in a numerical reliable way) and allows us to handle in a natural way the eigenvalue at infinity that originates the presence of spurious zeros.

[1] J.C. Zuniga and D. Henrion, A Toeplitz algorithm for polynomial J-spectral factorization, Automatica 42(7):1085-1093, 2006.

On Graph Products and the Resulting Laplacian Spectra

Sasmita Barik

Given two graphs G and H one can define several graph products. They give rise to many important classes of graphs. Imrich and Klavzar have introduced graph products and studied many graph invariants on the product graphs. Out of all graph products the cartesian product, categorical product, strong product and lexicographic product are probably the four most commonly used graph products. We investigate the Laplacian spectra of the product graphs obtained by the above mentioned four graph products. Some interesting results on the algebraic connectivity are also supplied for some special classes of graphs. We prove some results that help us to construct new Laplacian integral graphs from the known ones using the graph products.

Spectrally Arbitrary Zero-Nonzero Patterns over Finite Fields

Elizabeth Bodine

In this talk, we will investigate several zero-nonzero patterns over finite fields, and show over which fields certain patterns are spectrally arbitrary. We will explore the relationship between the number of nonzero entries and the size and characteristic of the field to develop some necessary conditions for a pattern to be spectrally arbitrary. We will look at some other necessary conditions that stem from field theory and number theory, as well as exploring some sufficient conditions and conjectures.

Lattice Basis Reduction and Polynomial Identities

Murray Bremner

The first part of this talk will describe the Hermite Normal Form (HNF) of an integer matrix; this is a generalization of the row canonical form in which divisions in the field of rational numbers are replaced by GCD computations in the ring of integers. I will explain how the HNF can be used to find solutions of systems of linear Diophantine equations, and discuss the problem of finding the "smallest" solutions. The second part of the talk will describe the LLL algorithm for lattice basis reduction (Lenstra, Lenstra and Lovasz, 1982), and mention its applications to computer algebra, cryptography and algebraic number theory. I will explain how the LLL algorithm can be used to find very "small" solutions to systems of linear Diophantine equations. The third part of the talk, based on joint work with Luiz A. Peresi (University of Sao Paulo), will describe the application of these techniques to the problem of finding "simple" polynomial identities for nonassociative structures.

Group Inverses of Matrices with Path Graphs

Minerva Catral

A simple formula for the group inverse of a 2×2 block matrix with a bipartite digraph is given in terms of the block matrices. This formula is used to give a graph-theoretic description of the group inverse of an irreducible tridiagonal matrix of odd order with zero diagonal (which is singular). An extension of the graph-theoretic description of the group inverse to singular matrices with tree graphs is conjectured. This is joint work with D. D. Olesky and P. van den Driessche.

Circulant Partial Hadamard Matrices

Robert Craigen

Almost 50 years ago Ryser conjectured that there are no Circulant Hadamard matrices of any order ≥ 4. Many research papers and several graduate theses later, the question is a long way from being settled, though much sophisticated machinery has been been harnessed.

Recently I began studying partial circulants, rectangular matrices obtained as the first few rows of circulant matrices. For which k and n does a partial circulant $k \times n$ (± 1)-matrix H exist satisfying $HH^T = nI$? If $k = n$ such a matrix would be a circulant Hadamard matrix. What behavior should a partial circulant display to qualify as a "near" circulant Hadamard matrix – that is, k as close to n as possible?

Considering the difficulty of Ryser's conjecture it is startling to find that a great deal can be established, using only elementary methods. We present very strong empirical and heuristic evidence in favor of the conjecture, unlike anything already known.

On the Fixed-point Iteration for the Matrix Equations $X \pm A^* X^{-1} A = I$.

Sandra Fital

The fixed-point iteration is a simple method for finding the maximal Hermitian positive definite solutions of the matrix equation $X \pm A^* X^{-1} A = I$ (the plus/minus equations). The convergence of this method may be very slow if the initial matrix is not chosen carefully. A strategy for choosing better initial matrices has been recently proposed by Ivanov, Hasanov and Uhlig. They proved that this strategy can improve the convergence in general and observed from numerical experiments that dramatic improvement happens for the plus equation with some matrices A. It turns out that the matrices A are normal for those examples. In this talk we show a result that explains the dramatic improvement in convergence for normal (and thus nearly normal) matrices for the plus equation. A similar result also holds for the minus equation.

Biorthogonal Cauchy Polynomials, Total Positivity and Matrix Models

Michael Gekhtman

The talk is based on an ongoing joint project with Marco Bertola (Concordia) and Jacek Szmigielski (Saskatoon). Motivated by a study of peakons - non-smooth soliton solutions appearing in certain nonlinear partial differential equations- we introduce a new class of polynomials biorthogonal with respect to a pairing between two Hilbert spaces with measures $d\alpha$, $d\beta$ on the positive semi-axis $\mathbb{R}_+$ coupled through the the Cauchy kernel $K(x, y) = \frac{1}{x+y}$. We establish fundamental properties of these polynomials: their zeros are interlaced, they satisfy four-term recurrence relations and generalized Christoffel-Darboux identities, they admit a characterization in terms of a 3 by 3 matrix Riemann-Hilbert problem. Moreover, a connection to certain novel two-matrix random matrix models will be pointed out.

Sign Patterns That Require Almost Unique Rank

Frank Hall

A sign pattern matrix is a matrix whose entries come from the set +, -, 0.The minimum rank mr(A) (maximum rank MR(A)) of a sign pattern matrix A is the minimum (maximum) of the ranks of the real matrices in the sign pattern class of A. Several results concerning sign patterns A that require almost unique rank, that is to say, the patterns A such that MR(A) = mr(A) + 1 are established. In particular, a complete characterization of these sign patterns is given. Further, the results on sign patterns that require almost unique rank are extended to sign patterns A for which the spread is d = MR(A) - mr(A).

Matrix Methods in Stability Theory

Olga Holtz

The univariate polynomial stability theory is intimately connected with many problems of matrix theory such as matrix factorizations, eigenvalue problems, and positivity properties of special classes of matrices. The intent of the talk is to present an overview of classical and new results of this type.

Mutually Unbiased Bases Arising from Weyl-Heisenberg Orbits

Mahdad Khatirinejad

A collection of $d + 1$ orthonormal bases in $\mathbb{C}^d$ such that for every distinct bases B and B' and every $\mathbf{u} \in B$ and $\mathbf{v} \in B'$ the equality $|\langle \mathbf{u}, \mathbf{v} \rangle|^2 = 1/d$ holds, is called *a complete set of mutually unbiased bases* or complete MUBs in short. The existence of complete MUBs is known for prime power dimensions and is still an unsolved problem for all the other dimensions. We study complete MUBs that are the union of a standard basis and an orbit of the Weyl-Heisenberg group. We present a characterization of such MUBs, show their existence for the prime power dimensions, and classify them for $d \leq 5$. Using this symmetry, and by minimizing a certain objective function, we are able to find numeric complete MUBs with high precision. MUBs have applications in digital communication and are also motivated by problems in quantum computing.

Two Small Theorems for Square Matrices Rotated a Quarter Turn

Peter Loly

The eight "phases" of a general square matrix resulting from rotations and reflections may be separated into two sets which interlace each other. The effects of rotation (e.g., a quarter turn) have the effect of moving between the two sets. We have found two theorems which describe: 1) the sign of the determinant (if non-singular), and, 2) a pair of characteristic equations (and thus eigenvalues). The reversal operator of all ones along the dexter diagonal is used in the proofs of these theorems. Simple second order examples suffice to illustrate the theorems, which emerged from a study of higher order matrices of magic squares.

Sufficient Conditions for Permutation Equivalence to a WHS-matrix

Dale Olesky

Square matrices with positive leading principal minors, called WHS-matrices (weak Hawkins-Simon), are considered in economics. Some sufficient conditions for a matrix to be a WHS-matrix after suitable row and/or column permutations have recently appeared in the literature. New and unified proofs and generalizations of some results to rectangular matrices are given. In particular, it is shown that if left multiplication of a rectangular matrix A by some nonnegative matrix is upper triangular with positive diagonal, then some row pemutation of A is a WHS-matrix. For a nonsingular A with either the first nonzero entry of each of its rows positive or the last nonzero entry of each column of A^{-1} positive, again some row permutation of A is a WHS-matrix. In addition, any rectangular full rank semipositive matrix is shown to be permutation equivalent to a WHS-matrix.

Some Results on Spectrally Arbitrary Sign Patterns

Rajesh Pereira

A sign pattern matrix S is an n by n matrix whose entries are in the set +,0,-. The sign pattern corresponding to S is the set of all n by n real matrices A such that $sgn(a_ij) = s_ij$ for all i and j. An n by n sign pattern is called spectrally arbitrary if any nth degree real monic polynomial is the characteristic polynomial of a matrix in the sign pattern. We will describe some new results which show that certain sign patterns are spectrally arbitrary if they contain certain types of nilpotent matrices.

Sums of Kronecker Products for Compound Magic Squares - Eigenproperties

Adam Rogers

Sums of Kronecker-Zeyfuss products are used to present a balanced view of two methods of compounding a pair of small natural magic squares to make larger ones of multiplicative order. At least one of these methods dates to before 1275 A.D., while the other may also be early, but so far has only been dated to 1869. If the pth order augmentation matrix of all 1's, $\mathbf{E}_p$, is reduced to $\bar{\mathbf{E}}_p = \frac{1}{p}$, we work with $\mathbf{Z}_{m,n} = \mathbf{X}_m \otimes \bar{\mathbf{E}}_n + \bar{\mathbf{E}}_m \otimes \mathbf{Y}_n$. Our treatment provides a definitive analysis of the eigenproperties of compound magic square matrices, which are always singular. The two methods transform into each other using the commutation matrix, and thus have the same rank. The rank of diagonable compound squares is the sum of the ranks of the pairs, less 1, by an argument using simple diagonable semi-magic squares, which encompasses magic simple diagonable magic squares with their additional diagonal properties. Some of the results may be extended to Latin squares and magic cubes.

Shape of a Drum, A Constructive Approach

Pappur Shivakumar

For the classical equation regarding the eigenvalues of the Laplacian, "Can you hear the shape of a drum?", the answer is known to be 'yes' for certain convex planar regions with an analytic boundary , although the answer is no for certain polygons. In this paper, a constructive approach is proposed for some simply connected real analytic plain domains. Our approach results in an infinite system of linear algebraic equations, coefficients of which are known polynomials of the eigenvalue. Expanding the resulting infinite determinant as a power series in the eigenvalues, we indicate how a pre-knowledge of the eigenvalues leads to the definition of the boundaries. Based on a previous paper by Yan Wu and the author, the ideas are applied to an elliptic membrane. Further, the power series can be used to find approximate numerical values of eigenvalues and corresponding eigenfunctions, using techniques like truncation, asymptotes, etc.

Some Comments on Graeco-Latin Squares and on Magic Squares, Illustrated with Playing Cards and Postage Stamps

George P.H.Styan

In this talk we comment on the Graeco-Latin squares and magic squares associated with the "Magic Card Puzzle", which is also known as "Bachet's square" [Henry Ernest Dudeney: *Amusements in Mathematics* (1970, p. 90)] and as "Ozanam's problem of the magic card square" [W. W. Rouse Ball & H. S. M. Coxeter: *Mathematical Recreations & Essays*, 13th edition, Dover, New York (1987, p. 191)]; see also Jacques Ozanam: *Récreations mathématiques et physiques ... avec l'explication des tours de gibecière*, Nouvelle édition, tome quatrième, Claude Jombert, Paris (1725, p. 434 & Fig. 35, Pl. 12)]; we also consider "Euler's Problem of the 36 officers".

We will present our comments from an historical perspective, illustrated with images of playing cards and postage stamps. [Joint work with Christian Boyer (Enghien-les-Bains, France) and Ka Lok Chu (Dawson College, Westmount (Québec), Canada).]

Bounds For the Perron Root Using Max Eigenvalues

Pauline van den Driessche

Using the techniques of max algebra, a new proof of Al'pin's lower and upper bounds for the Perron root of a nonnegative matrix is given. The bounds depend on the row sums of the matrix and its directed graph. If the matrix has zero main diagonal entries, then these bounds may improve the classical row sum bounds. This is illustrated by a generalized tournament matrix.

Understanding the QR Algorithm, Part IX

David Watkins

After some 45 years of use, the QR algorithm is still the most important method for computing the complete set of eigenvalues and eigenvectors of a matrix. How should we understand the QR algorithm and teach it to our students? There is a huge gap between the most basic version of the algorithm and the versions that are actually used in practice. Should we just cover the basic version and leave it at that? Should we start with the basic version and work by stages to the implicitly-shifted multistep QR algorithms that are used in practice? In this talk we outline a pedagogical pathway that leads directly to the practical algorithm, bypassing the basic QR algorithm completely.

Spectrally Arbitrary Zero-Nonzero Patterns

Amy Yielding

In this talk we will determine the number of nonzero entires needed to require a pattern to be a spectrally arbitrary pattern. We will begin by looking at irreducible patterns with 0 or 1 transversal, then extend this result to reducible patterns.

3 Poster Abstracts (alphabetical)

On Reducible Matrix Patterns

Michael Cavers

An n by n nonzero (resp. sign) pattern $\mathcal{A}$ is a matrix with entries in $\{*, 0\}$ (resp. $\{+, -, 0\}$), where $*$ denotes a nonzero real number. We say $\mathcal{A}$ is inertially arbitrary if each ordered triple (a_1, a_2, a_3) of nonnegative integers with $a_1 + a_2 + a_3 = n$ is the inertia of a matrix with nonzero (resp. sign) pattern $\mathcal{A}$. Sufficient conditions on $\mathcal{A}$ and $\mathcal{B}$ for the direct sum $\mathcal{A} \oplus \mathcal{B}$ to be inertially arbitrary are presented. Smallest order inertially arbitrary patterns $\mathcal{A} \oplus \mathcal{B}$, where $\mathcal{A}$ and $\mathcal{B}$ are not necessarily inertially arbitrary are looked at.

The Dual of a Graph

Ryan Tifenbach

We consider various problems involving the dual of a graph, a type of graph-inverse. We present various families of graphs that possess duals, and characterizations of classes of graphs whose dual possess certain required properties.

4 Participants

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