Waves and Instabilities in idealized model convective parametrization

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Outline

- Single layer models
  - CISK-like instability
  - Stability of adjustment scheme
- One-and-a-half layer models: WISHE waves (LPA, ICAPE, Quasi-Equilibrium Schemes)
- 2-and-a-half-layer models: Stratiform instability
- **Multicloud models**

Idealized convergence model: CISK-like mode

Consider a dynamical model for the topical atmosphere with a crude vertical resolution reduced to the first vertical baroclinic mode. For simplicity we use the linear shallow water equations without rotation (beta is set to zero). Such a model is believed to be suitable for the study of the large scale response of the tropical atmosphere to cumulus convection (Gill 1980, etc.).

$$\begin{aligned} u_t - \theta_x &= 0 \\ \theta_x - u_x &= Q \end{aligned} \tag{1}$$

Here u is the horizontal velocity, θ is the potential temperature, and Q is the condensational or convective heating.

The main problem in convective parametrization is to come up with a closure for Q a function of the large scale variables, u, θ.

The CISK theory, pioneered by Charney and Eliassen (1964), assumes that the convective heating is proportional to the low-level convergence of moisture

$$Q \propto -\mathrm{div}(q\mathbf{v})_{z=z_l},$$

which implies heating in regions of large scale convergence and cooling in divergence regions. Predefined profiles of the heating field mimicking various cumulus cloud types are also assumed. Here z_l is some fixed height around the cloud base and q is the specific humidity. In our idealized linear model (**??**), the q equation decouples and thus ignored for simplicity. Our heating profile is the one representing deep convective clouds, which is consistent with the first baroclinic mode approximation utilized here. We set

$$Q = -\alpha u_x. \tag{2}$$

Combining (**??**) and (**??**) yields the following wave equation for θ alone

$$\theta_{tt} = (1 - \alpha)\theta_{xx}.$$

When $\alpha \leq 1$, we have two stable waves travelling in the opposite directions with the reduced speed $c_r = \pm\sqrt{1-\alpha}$: $\theta_\pm = \theta_0(x \pm c_r t)$. **Whereas when $\alpha > 1$ the system becomes unstable and we have two standing modes, one grows exponentially and the other is damped**:

$$\theta(x,t) = e^{\pm\sqrt{\alpha-1}kt}e^{ikx}.$$

The growth rate of the unstable mode increases linearly with the wavenumber, k. Shorter wavelengths grow faster than longer ones, no scale selection. This is typical to wave-CISK. Thus, despite the crudeness of this model, which is by many means far from being 'realistic', it exhibits quite well one of the most salient features of CISK models.

To correct for this catastrophic instability at small/unresolved scale and provide a scale selection CISK, at least two different modifications are available:

- Lagged wave-CISK, Davies (1979)
- Frictional or boundary layer wave-CISK, Wang (1988)

We end this section by rephrasing Charney and Eliassen (1969):

"...This suggests that we should look upon the pre-hurricane depression and the cumulus cell not as competing for the same energy..., rather we should consider the two as supporting one another–the cumulus cell by supplying the heat energy for driving the depression and the depression by producing the low-level convergence of moisture into the cumulus cell. ...this type of interaction does lead to a large scale self amplification, which we may call *conditional instability of the second kind* to contrast with the conditional instability for small-scale cumulus convection"

A stable adjustment scheme

The quasi-equilibrium (Arakawa and Shubert 1974) philosophy assumes that on average the tropical atmosphere remains in a stable state. Convection adjusts the atmosphere to a state of equilibrium (Betts).

... the simplest such model is the adjustement scheme of Betts and Miller (1986). Temperature and moisture adjust dynamically to a reference (stable) state. We consider a simplified version (used by Frierson et al., etc. for moisture front propagation)

$$\begin{aligned} u_t - \theta_x &= 0 \\ \theta_t - u_x &= Q_c^+ - Q_R \\ q_t + Q u_x &= -P + E \end{aligned} \tag{3}$$

Here $Q_c = \frac{1}{\tau_q}(q - \hat{q}) - \frac{1}{\tau_\theta}(\theta - \hat{\theta})$ is the convective heating, Q_R is a prescribed constant radiative cooling, $P = Q_c^+$ is the pricipitation rate, and $E = \frac{1}{\tau_e}(q_s^* - q)$ is the evapration rate.

Linearized Eqns about a Radiative convective equilibrium (RCE).

An RCE is a homogeneous stady state solution for the PDE system where convective forcing $\bar{Q}_c$ balances the radiative cooling. We consider a small perturbation about this RCE solution and write the linearized eqns for the perturbation.

$$\begin{aligned} &u_t - \theta_x = 0 \\ &\theta_t - u_x = \frac{1}{\tau_q} q - \frac{1}{\tau_\theta}\theta \\ &q_t + Q u_x = -\left(\frac{1}{\tau_q} + \frac{1}{\tau_e}\right) q + \frac{1}{\tau_\theta}\theta \end{aligned} \tag{4}$$

Look for plane wave solutions

$$\begin{pmatrix} u \\ \theta \\ q \end{pmatrix} = U e^{i(kx - \omega t)}$$

k is zonal wavenumber, $\omega \equiv \omega(k)$ (dispersion relation) is the genralized phase, with

$Im(\omega) \equiv$ growth rate, $Re(\omega)/k \equiv$ phase speed.

Linear eigenvalue problem

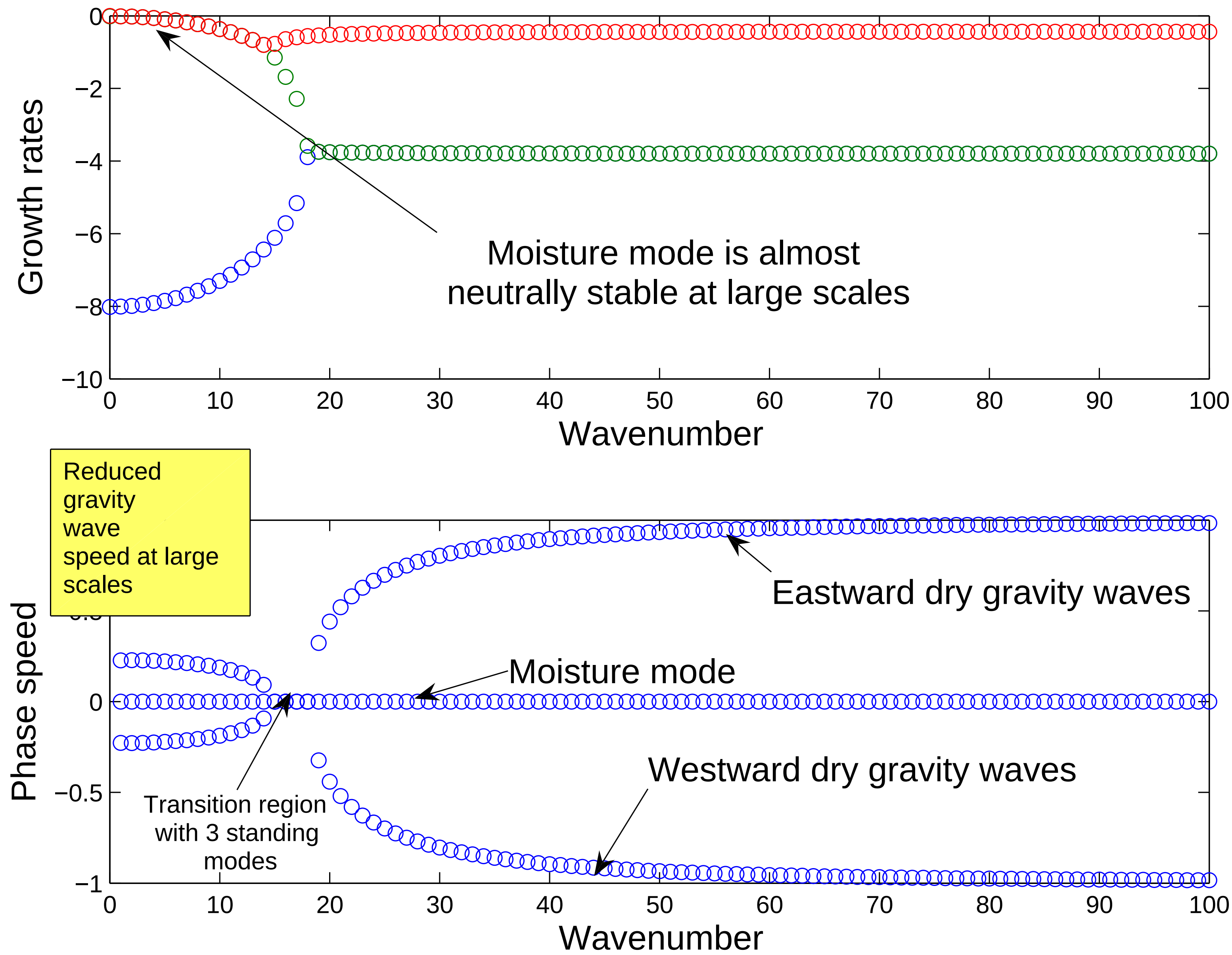
$$-i\omega U + ikAU = BU,$$

or

$$\omega U = (kA + iB)U$$

with

$$A = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ Q & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1/\tau_\theta & 1/\tau_q \\ 0 & 1/\tau_\theta & -(1/\tau_q + 1/\tau_e) \end{bmatrix}$$


Growth rates
0
−2
−4
−6
−8
−10
0 10 20 30 40 50 60 70 80 90 100
Wavenumber
Moisture mode is almost
neutrally stable at large scales
Reduced
gravity
wave
speed at large
scales
Phase speed
0
−0.5
−1
Eastward dry gravity waves
Moisture mode
Westward dry gravity waves
Transition region
with 3 standing
modes
Wavenumber

Adjustement scheme has

- 2 damped gravity waves moving in the opposite directions (with perfect symmetry between east and west)
- The propagation speed of the waves approaches ± 50 m/s (the speed of the dry gravity) at small scales (large k)
- A damped and standing mode often called *the moisture mode* because induced by the moisture equation
- Note a significantly reduced wave speed at large scales
- A transition region with three standing modes
- The system is stable (growth≤ 0) and strongly damped except for the moisture mode which is almost neutral at large scales

We need an external mechanisms to make such scheme unstable. In quasi-equilibrium schemes nonlinear surface or radiative fluxes are often used to trigger 'convective' instabilities.

WISHE: wind induced surface heat exchange (Emanuel, 1987) is one of them.

WISHE waves in One-and-a-half layer models

We consider a one-and-a-half layer model for the tropical atmosphere: A full dynamical layer for the free troposphere on top of the thin well mixed boundary layer near the sea surface. The boundary layer is passive except for time variations in θ_{eb}. Following Majda and Shefter (JAS 2001), we have

$$\begin{aligned} u_t - yv - \theta_x &= -\frac{C_D}{h}(|\mathbf{v}|)u \\ v_t + yu - \theta_y &= -\frac{C_D}{h}(|\mathbf{v}|)v \\ \theta_t - \mathrm{div}(\mathbf{v}) &= Q_c - Q_R \\ \frac{\partial \theta_{em}}{\partial t} &= \frac{1}{H}D - Q_R \\ \frac{\partial \theta_{eb}}{\partial t} &= \frac{1}{h}E - \frac{1}{h}D. \end{aligned} \tag{5}$$

In (**??**), $\mathbf{v} = (u, v)$ is the horizontal velocity field and θ the potential temperature while θ_{eb} and θ_{em} are respectively the equivalent potential temperatures in the thin boundary layer and the middle of the troposphere. The term $\frac{C_D}{h}(|\mathbf{v}|)$ is a the coeffient of nonlinear momentum drag, Q_c is the convective heating, $Q_R = Q_R^0 + \frac{1}{\tau_R}\theta$ is the long-wave radiative cooling,

$$E = C_\theta(|\mathbf{v}|)(\theta_{eb}^* - \theta_{eb})$$

is the evaporation from the ocean surface, where θ_{eb}^* is the saturation θ_e in the boundary layer, and D is the downdrafts. Note that downdrafts dry and cool the boundary layer and warm and moisten the upper troposphere by the same amount. As a result the system conserves the vertical integrated moist static energy, namely, the quantity $\theta_{ez} = \theta_{em} + \frac{H}{h}\theta eb$ is conserved when the external effect, Q_R and E are set to zero. Note the WISHE effect comes from the term $C_\theta(|\mathbf{v}|)$ impling an amplification of the surface evaporation with the increasing amplitude of the wind.

Within the large scale GCM grid box, decompose the vertical velocity into a clrear sky and within cloud comtributions:

$$w = (1 - \sigma_c)w_e + \sigma_c w_c$$

where w_c is the average vertical velocity within the cloud (convective, upward) and w_e is the vertical velocity in the environment outside the cloud with σ_c is the area fraction of the cloudy region (assumed small compared to the grid box).

We assume that the convective heating is proportional to the upward velocity within the clouds, w_c, which is an unresolved variable (unlike the CISK theory).

$$Q_c = \frac{\bar{\alpha}}{H_m}\sigma_c w_c$$

Note the qunatity $\sigma_c w_c$ is also interpreted as the convective mass flux. Here $\bar{\alpha}$ is a dimensional constant and H_m is the middle tropospheric height.

The downdrafts have an environmental and a convective components

$$D = -[(1-\sigma_c)w_e^- - \sigma w_d](\theta_{eb} - \theta_{em})$$

$$w_d = \frac{1-\epsilon_p}{\epsilon_p} w_c; \quad \epsilon_p : \text{ precipitation efficiency.}$$

$$(1-\sigma_c)w_e = w - \sigma_c w_c = H_m \text{div}(\mathbf{v}) - \sigma_c w_c$$

$$D(|\mathbf{v}|) = (u_0^2 + |\mathbf{v}|^2)^{1/2}$$

u_0 size of turbulent fluctuations.

Note: $X^+ = \max(X,0), \quad X^- = \min(X,0)$

Convective closures:

1. Lagrange parcel adjustment (LPA)

$$\frac{\partial w_c}{\partial t} = (\Gamma B - \frac{w_c^2}{2H})\mathcal{H}(w_c), \quad \mathcal{H}(w_c) = \begin{cases} 1 & \text{if } w_c > 0 \\ 0 & \text{otherwise} \end{cases}$$

$\Gamma B = \Gamma(\theta_{eb} - \theta^*_{em})$ represents fluctuations in the convective available potential energy (CAPE), the vertical integral of the positive part of the convective buoyancy force.

$w_c^2/2$ is the kenitic energy of the convective parcel, which adjusts dynamically to positive CAPE fluctuations.

If $B > 0$ then w_c increases and when $B < 0$ w_c decreases untill it reaches zero.

(note $\theta^*_{em} \approx \gamma\theta$)

2. Instantaneous adjustement to CAPE, ICAPE

$$\frac{w_c^2}{2H} = \Gamma(\theta_{eb} - \theta_{eb}^*)^+$$

3. Quasi-Equilibrium scheme Assume $B \equiv 0$ at all time; convection mantains the tropical atmosphere in a stable equilibrium state at large scale. The system is reduced to 4 prognostic equations

$$\Longrightarrow \ \theta_{eb} = \gamma\theta$$

In which case the two equations for θ_{eb} and θ can used to derive a prognostic equation for w_c.

$$\frac{1}{h}E - \frac{1}{h}D = \gamma\frac{\bar{\alpha}}{H_m}\sigma_c w_c - \gamma Q_R + \mathrm{div}(|\mathbf{v}|)$$

Here we have only three prognostic equations

RCE AND LINEAR SYSTEMS

Again, we consider the simple case without rotation. The RCE solution for each system is derived in a similar way as above, however here we assume a non zero baroclinic mean, $\bar{u}$, a horizontal baroclinic shear.

In particular the drag coefficient induces a non trivial term into the linearized evaporative forcing involving the wind perturbations, which implies a large scale feedback of the wave into the surface evaporation.

$$E = \mathcal{D}(|\bar{u}+u'|)(\theta^*_{eb} - \bar{\theta}_{eb} - \theta'_{eb}) = (u_0^2 + (\bar{u}+u')^2)^{1/2}(\theta^*_{eb} - \bar{\theta}_{eb} - \theta'_{eb})$$

$$\approx (u_0^2 + \bar{u}^2)^{1/2}(\theta^*_{eb} - \bar{\theta}_{eb}) - (u_0^2 + \bar{u}^2)^{1/2}\theta'_{eb} + \frac{\bar{u}}{(u_0^2 + \bar{u}^2)^{1/2}}(\theta^*_{eb} - \bar{\theta}_{eb})u'$$

Note we have an enhancement in surface evaporation if $\bar{u}$ and u' have the same sign. Therefore easterly mean wind will favor eastward moving waves while westerly winds favor westward moving waves.

Some linear stability results from Majda and Shefter

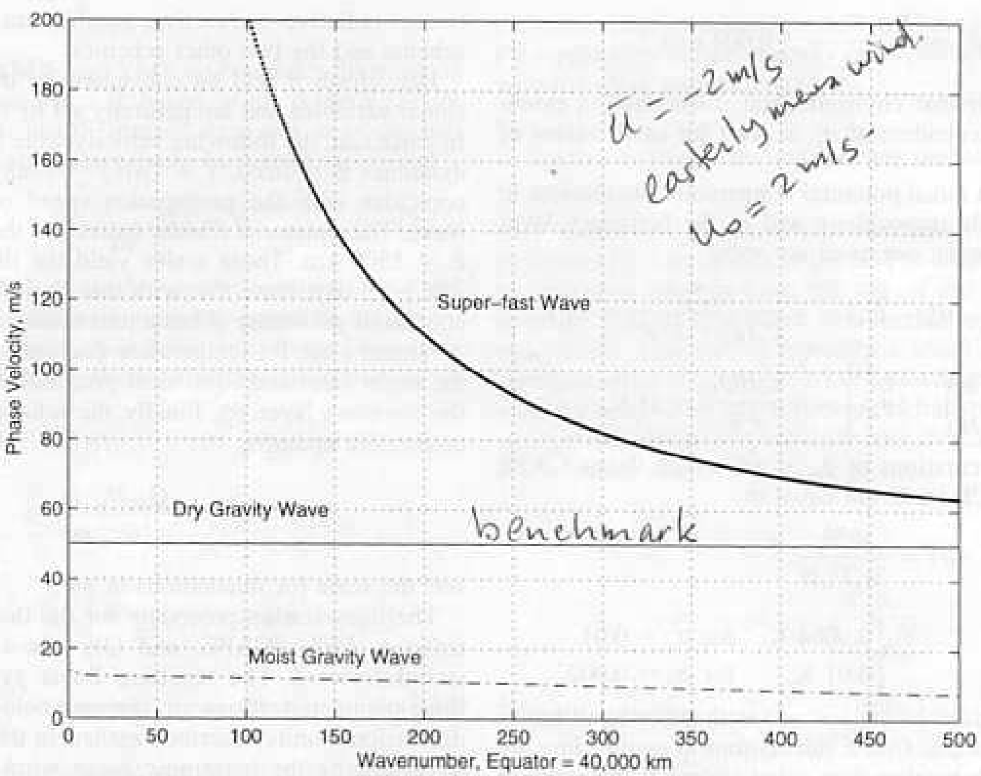


FIG. 1. Phase velocity diagram for moist eastward superfast waves (thick solid line) and moist eastward gravity waves (dash-dotted line) in the LPA system without rotation. The speed of dry gravity waves is indicated by a thin solid line.

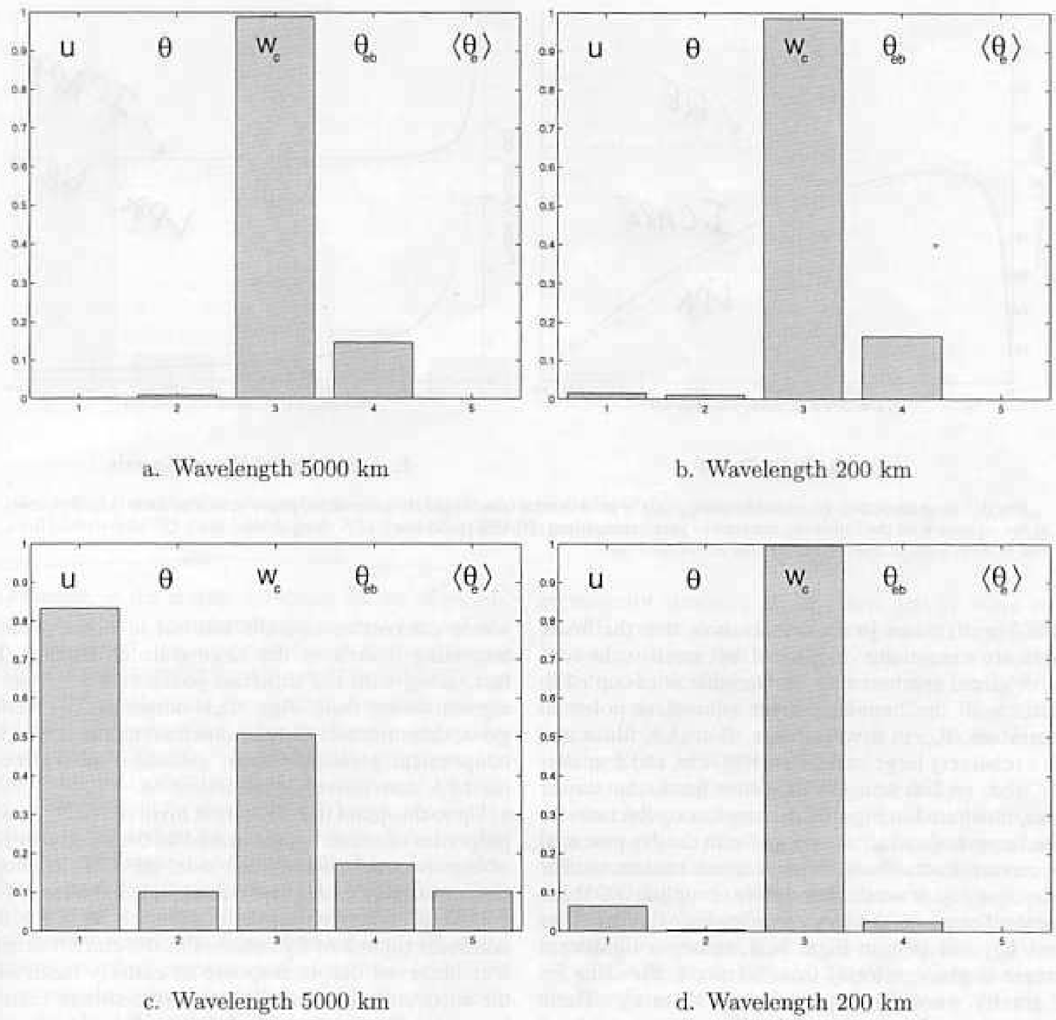
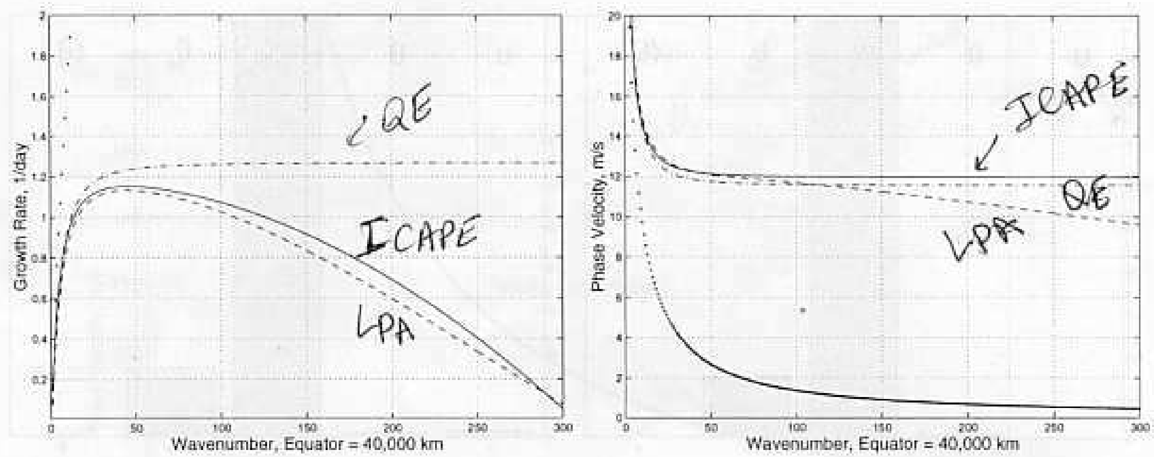


Fig. 2. Eigenvector structure of moist eastward superfast waves, (a) and (b), and moist eastward gravity waves, (c) and (d), for the LPA system without rotation. The wavelengths are 5000 km for (a) and (c), and 200 km for (b) and (d).



a. Growth Rates b. Generalized Phase Speeds

FIG. 3. Linear properties for unstable modes: (a) the growth rates (day^{-1}) and (b) generalized phase velocities ($m\ s^{-1}$) for dynamics at the equator with the following convective parameterizations: ICAPE (solid line), LPA (long-dashed line), QE (dot–dashed line), and ICAPE without convective downdrafts (dotted line).

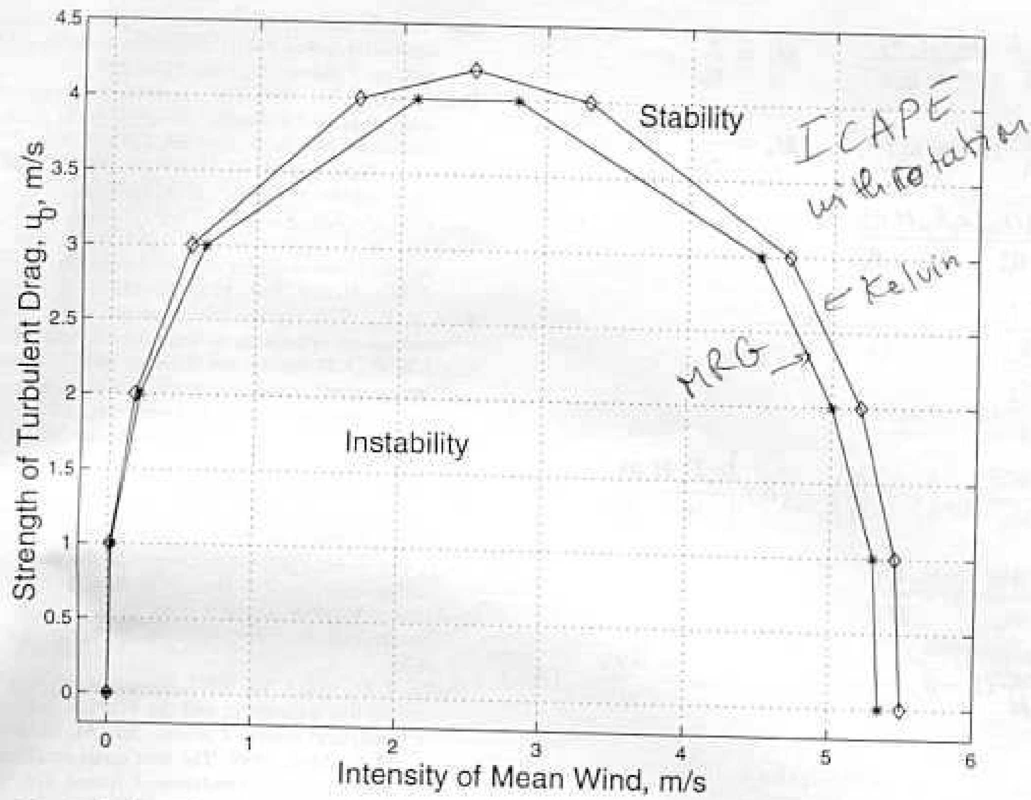


Fig. 8. Regions of linear stability and instability for the meridionally truncated ICAPE system forced by easterly barotropic mean flow. The parameter space includes the strength of barotropic mean flow, $|\bar{u}|$, and the strength of turbulent drag, expressed via mean velocity fluctuations, u_0. The top and bottom curves represent the critical values of mean wind required to generated instability of moist Kelvin (diamonds) and moist mixed Rossby–gravity (asterisks) waves, for various values of u_0.

A few remarks

- LPA: 2 moist gravity waves moving with a reduced speed of about 10-12 m/s, unstable at large scales–scale selection 2 superfast waves which become unstable at small scales when $\bar{u}, u_0$ are small. Those are unphysical (spurious) waves generated by the finite time convective adjustement scheme, and a fifth mode which is standing and almost neutral (reminiscent to the moisture mode seen in previous adjustment scheme)
- ICAPE eliminates the superfast waves and replaces them by a damped stading mode. The two moist gravity waves are robust.
- The QE scheme has a similar pair of moist gravity waves plus a satnding mode. The growth of the moist gravity waves approaches, from below, a positive constant value at small scales–no scale selection. It is not as catastrophic as CISK but the prefered wavenumbers of instability are at the grid size.
- WIISHE evidence (fig. 8 of MS): ... those are WISHE waves triggered by the enhancement of surface evaporation due to the wind fluctuations. Whether this is physical or not is very questionable... the MJO has a stronger westerly wind to west than easterly wind to the east.

Stratiform instability

Mapes 2000, Majda and Shefter 2001.

Observed Features of Superclusters

(1) Eastward phase speed: roughly 15 m s^{-1}.

(2) Horizontal wavelength of about 2000 to 5000 km.

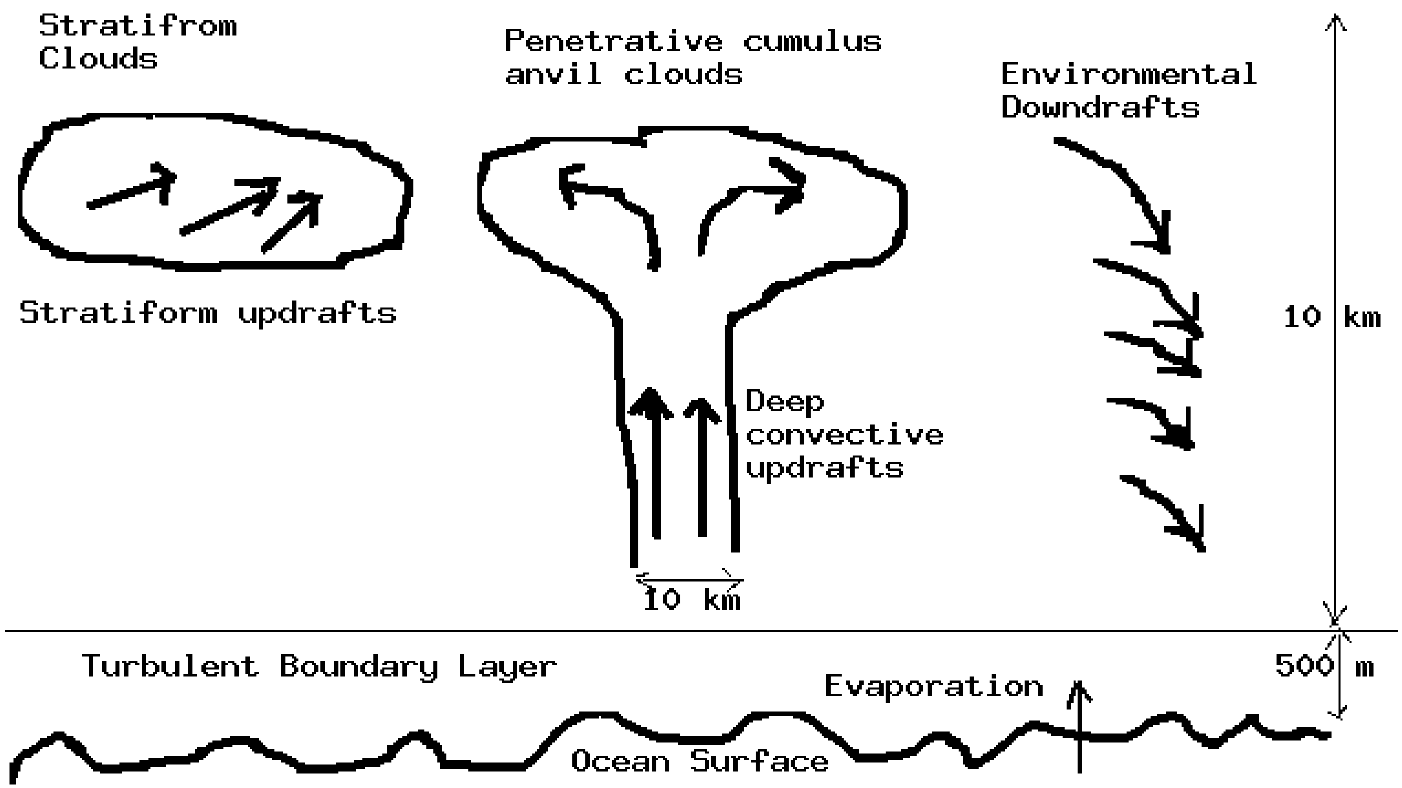
(3) Anomalously cold temperatures in lower troposphere (below 500 hPa) and warm in the upper troposphere (500-250 hPa) within and often leading the region of heating and strong updrafts.

(4) The wind and pressure have upward-westward tilt in lower troposphere (below 250 hPa) inducing a boomerang shape in the vertical.

(5) Anomalous increases in θ_{eb}, i.e, CAPE, shallow convection, and low level moisture convergence lead the wave.

(6) Trailing part dominated by stratiform precipitation.

The model convective parametrization with two heating modes:

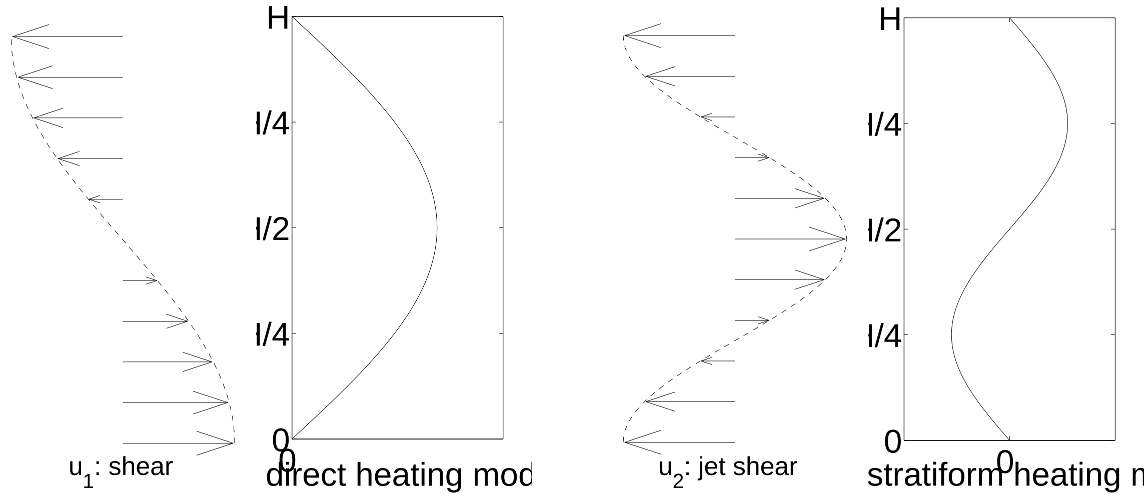


1) Galerkin truncation: Project primitive (β-plane) equations on 1st and 2nd vertical baroclinic modes

$$G_1(z) = \cos(\frac{\pi z}{H_T}), \quad G_2(z) = -\cos(\frac{2\pi z}{H_T})$$

$$\begin{pmatrix} \vec{v} \\ p \end{pmatrix} = \begin{pmatrix} \vec{v}_1 \\ p_1 \end{pmatrix} G_1(z) + \begin{pmatrix} \vec{v}_2 \\ p_2 \end{pmatrix} G_2(z)$$

$$\begin{pmatrix} w \\ \theta \end{pmatrix} = \begin{pmatrix} \vec{w}_1 \\ \theta_1 \end{pmatrix} \bar{\lambda}\bar{H}(-G_1') + \begin{pmatrix} w_2 \\ \theta_2 \end{pmatrix} \bar{\lambda}\bar{H}(-G_2')$$



2) Two coupled shallow water systems:

$$\begin{cases} \frac{D\vec{v}_1}{Dt} - \bar{\bar{\alpha}}\nabla_H\theta_1 + \beta y\vec{v}_1^{\perp} = -C_{D,1}\vec{v}_1 - \tau_D^{-1}\vec{v}_1 \\ \frac{D\theta_1}{Dt} - \bar{\alpha}\mathrm{div}_H\vec{v}_1 = S_1, \end{cases} \quad (6)$$

$$\begin{cases} \frac{D\vec{v}_2}{Dt} - \bar{\bar{\alpha}}\nabla_H\theta_2 + \beta y\vec{v}_2^{\perp} = -C_{D,2}\vec{v}_2 - \tau_D^{-1}\vec{v}_2 \\ \frac{D\theta_2}{Dt} - \frac{\bar{\alpha}}{4}\mathrm{div}_H\vec{v}_2 = S_2, \end{cases} \quad (7)$$

$(\bar{\alpha}\bar{\bar{\alpha}})^{1/2} \approx 50 \text{ m s}^{-1}$,

$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{\bar{v}}_1 \cdot \nabla_H$

S_1, S_2 represent convective heating and radiative cooling.

$S_1 = q_1 + Q_{R,1}, \quad S_2 = q_2 + Q_{R,2}$

3) Heating and Radiative Cooling:

$S_1 = q_1 + Q_{R,1}, \quad S_2 = q_2 + Q_{R,2}$

$Q_{R,1} = \frac{1}{1+s} Q_{R_0} - \frac{1}{1+s}\frac{1}{\tau_R}\theta_1,$ Newtonian cooling

$Q_{R,2} = \frac{s}{1+s} Q_{R_0} - \frac{s}{1+s}\frac{1}{\tau_R}\theta_2$

Direct-deep convective heating is proportional to CAPE:

$$q_1 = \sigma_c \frac{\bar{\alpha}}{H_m} (\mathrm{CAPE}^+)^{1/2}$$

$\mathrm{CAPE} = 2Hc_p\Gamma_m \frac{\theta_{eb} - \theta^*_{em}}{\theta_0}$

$\theta^*_{em} \approx \gamma\bar{\lambda}\theta_1 - \alpha_2\gamma\bar{\lambda}\theta_2, \quad \gamma = \frac{\Gamma_d}{\Gamma_m} = 1.7$

$w_c = (\mathrm{CAPE}^+)^{1/2},$ deep convective updraft velocity

σ_c: area fraction of deep convection, $0.001 \le \sigma_c \le 0.01$,

4) Stratiform heating and boundary layer θ_e:

$$\frac{\partial q_2}{\partial t} = \frac{1}{\tau_s}(sq_1 - q_2)$$
$$h\frac{\partial \theta_{eb}}{\partial t} = -D + E \quad (\theta_e = \theta + \tilde{K}r). \tag{8}$$

D downward mass flux (drying and cooling)

E evaporation from the ocean surface (heating and moistening)

5) Downdrafts:

$$D = \left[m_+ + (\sigma_c - 1)w_e^-\right](\theta_{eb} - \bar{\theta}_{em})$$

$m_+ = \frac{1-\Lambda}{\Lambda}[\mu m_s + (1-\mu)m_c]$, downward mass flux due to precipitation: $\Lambda = 0.9$

$m_c = \sigma_c w_c = \frac{H_m}{\bar{\alpha}} q_1$: deep convective mass flux

$m_s = \frac{H_m}{s\bar{\alpha}} q_2$: stratiform mass flux

$(1-\sigma_c)w_e = w_{e,1} - \alpha_2 w_{e,2}$: environmental mass flux

$w_{e,1} = -(m_c + H_m \mathrm{div}\vec{v}_1)$,

$w_{e,2} = -(\frac{H_m q_2}{\bar{\alpha}_2} + \frac{H_m}{4}\mathrm{div}\vec{v}_2)$

6) Surface fluxes (WISHE):
Evaporative heating: $E = C_\theta^0 \mathcal{D}(\mathbf{v})(\theta_{eb}^* - \theta_{eb})$

Turbulent momentum drag:

$C_D(\vec{v}) = h^{-1} C_D^0 \mathcal{D}(\mathbf{v})$, $\mathcal{D}(\mathbf{v}) = [u_0^2 + |\vec{v}_1 - b\vec{v}_2|^2]^{1/2}$

$$C_{D,1} = \frac{1}{1+b} C_D(\vec{v}), \quad C_{D,2} = \frac{b+\delta_0(b)}{1+b} C_D(\vec{v}),$$

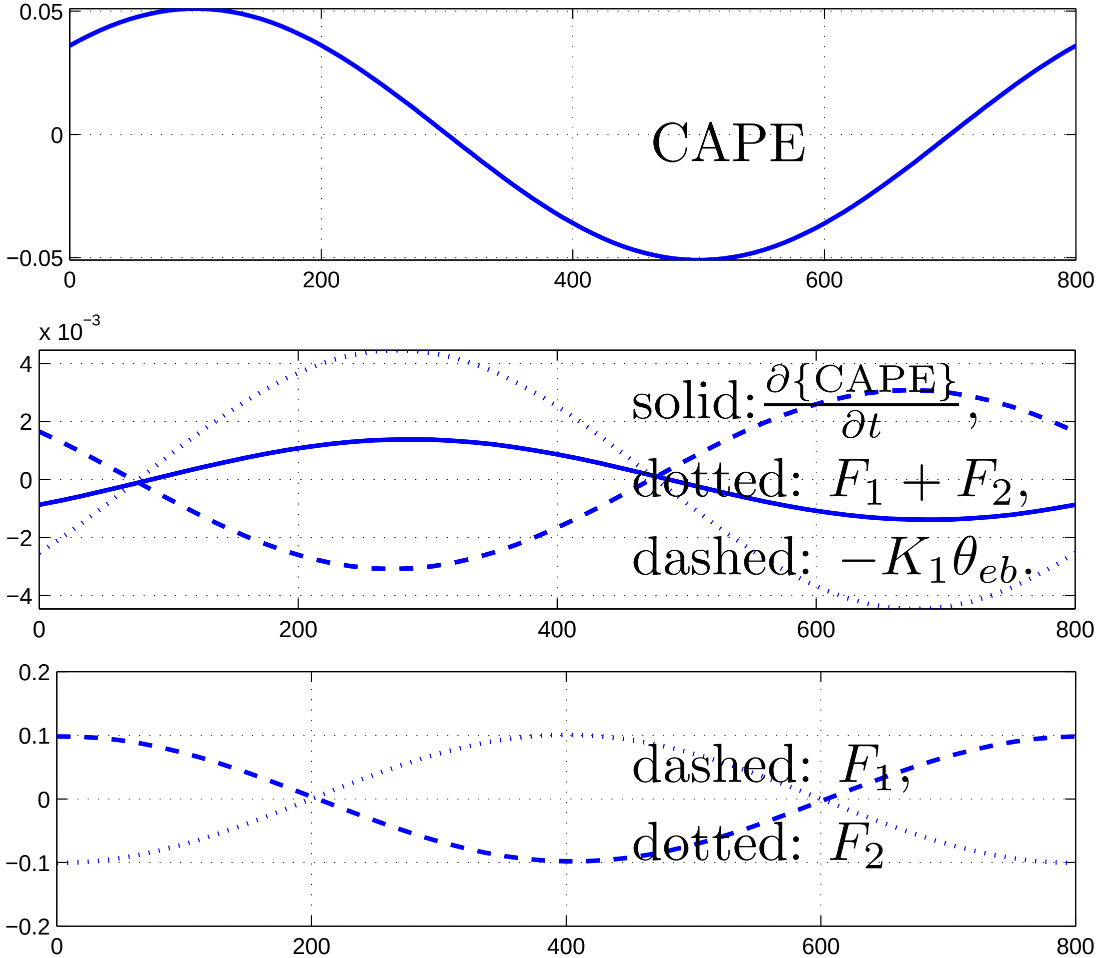
Linear stratiform instability:

- First observed and analysed by Majda-Shefter (JAS, 2001), Various observed aspects of Kelvin waves are reproduced. σ_c is a key instability parameter:
  - when $\sigma_c = .0014$ scale selective instability; peaks at around 1500 km;
  - when $\sigma_c = .01$: instability amplifies and extends to small scales;
  - system is stable for $\sigma_c \leq 0.001$

  wavelength 1500 km $\longleftrightarrow$ 15 m s^{-1}, vertical tilt.

- M-K-K-S-S (2002), simplified version of MS model ($\alpha_2 = b = 0, \mu \neq 0$):
  Linear waves have the same features as in M-S 2001.

Stratiform instability mechanism



Negative anomalies in m_s lead increase in θ_{eb}, thus (re)build CAPE:

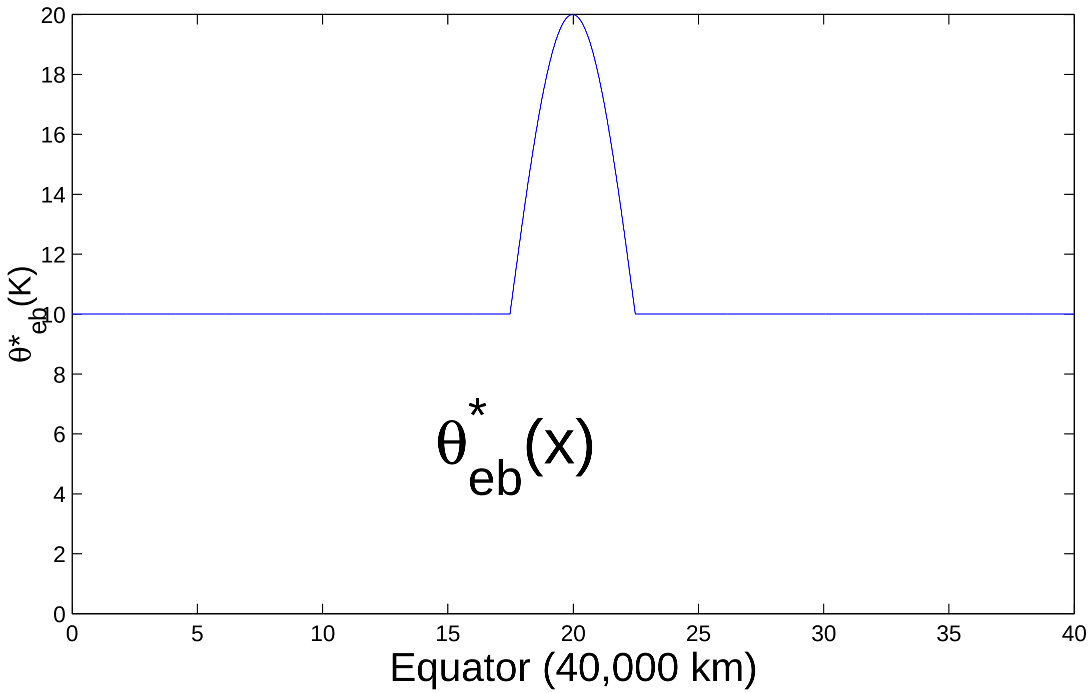
$$\frac{\partial\{\text{CAPE}\}}{\partial t} = F_1 + {\color{red}F_2} - K_\theta \theta_{eb}$$

with $K_\theta > 0$, F_1 forcing of CAPE due to first baroclinic variables (u_1, θ_1), F_2 forcing by second baroclinic variables (u_2, θ_2, q_2) (Here with $\alpha_2 = 0, b = 0$, $F_2 = -K_2 q_2, K_2 > 0$) Note regions where $\frac{\partial\{\text{CAPE}\}}{\partial t} > 0 \iff$ regions where $F_1 < 0, -K_\theta\theta_{eb} < 0$, ${\color{red}-K_2 q_2 > 0}$

Nonlinear Simulations of Superclusters

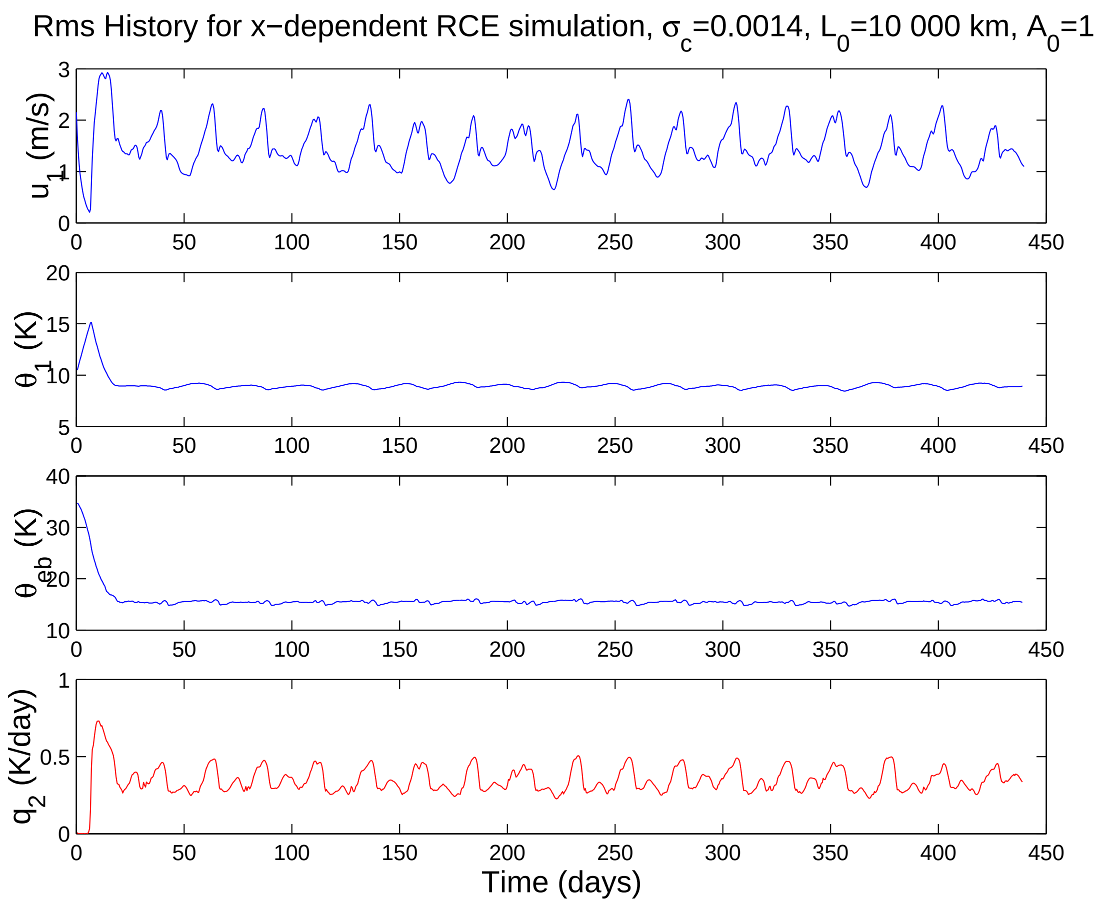
Walker circulation set-up:

mimicking the Indian Ocean/Western Pacific warm pool

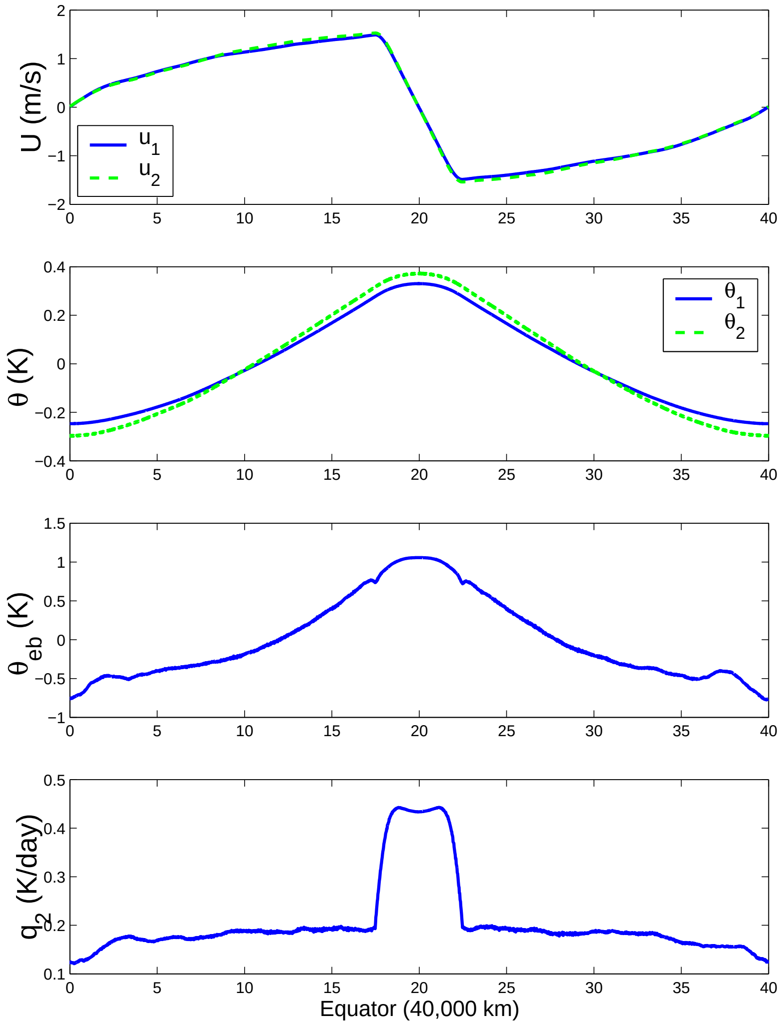


- periodic geometry, $\Delta x = 40$ km, $\sigma_c = 0.0014$, fixed
- initial data: RCE + small random perturbation
- Integrate to statistical equilibrium.

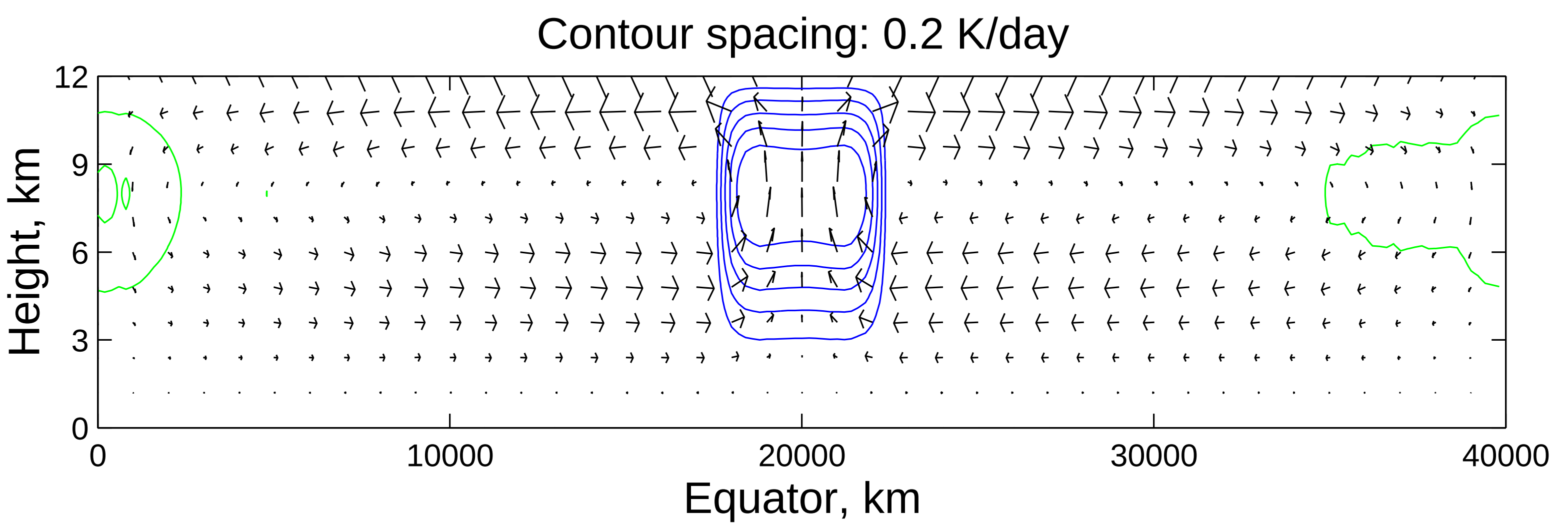
Convergence to statistical steady state



Climatology consistent with the imposed θ_{eb}^*

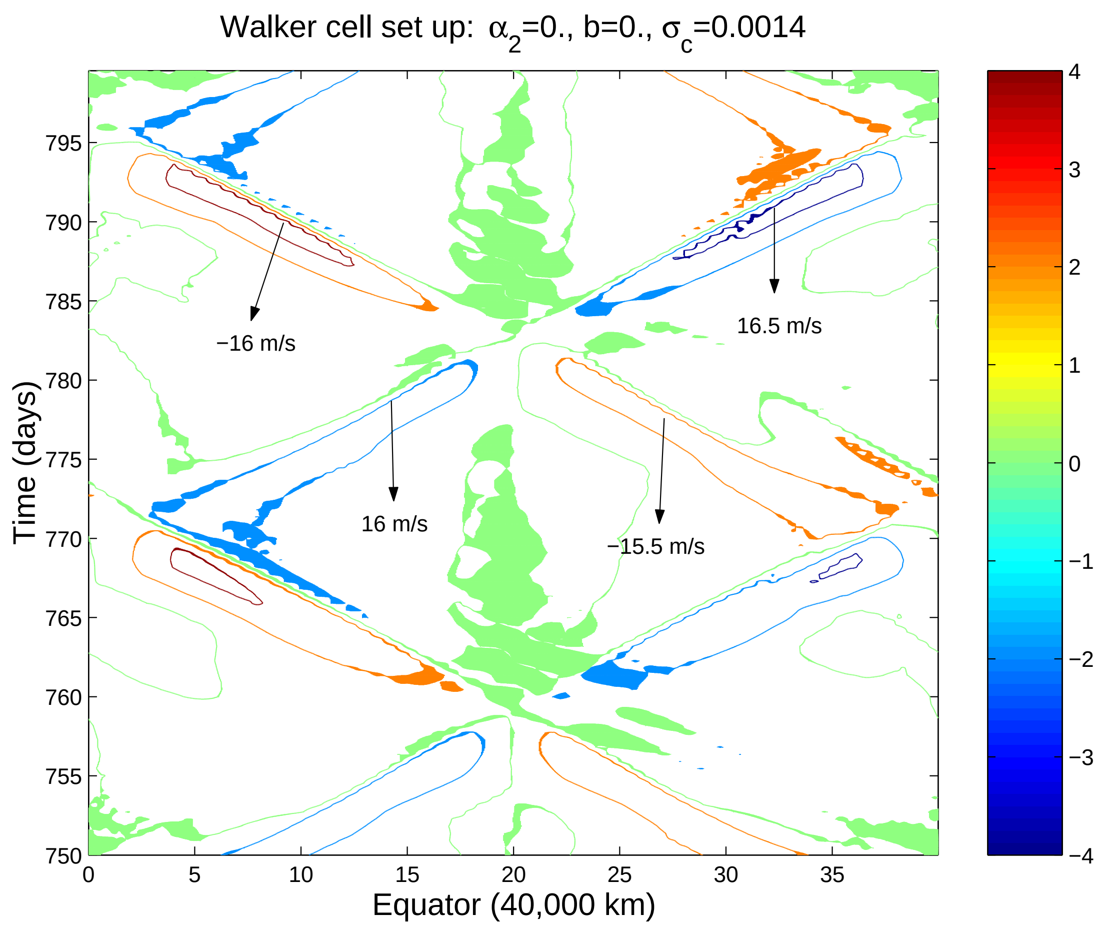


Mean heating and velocity profile



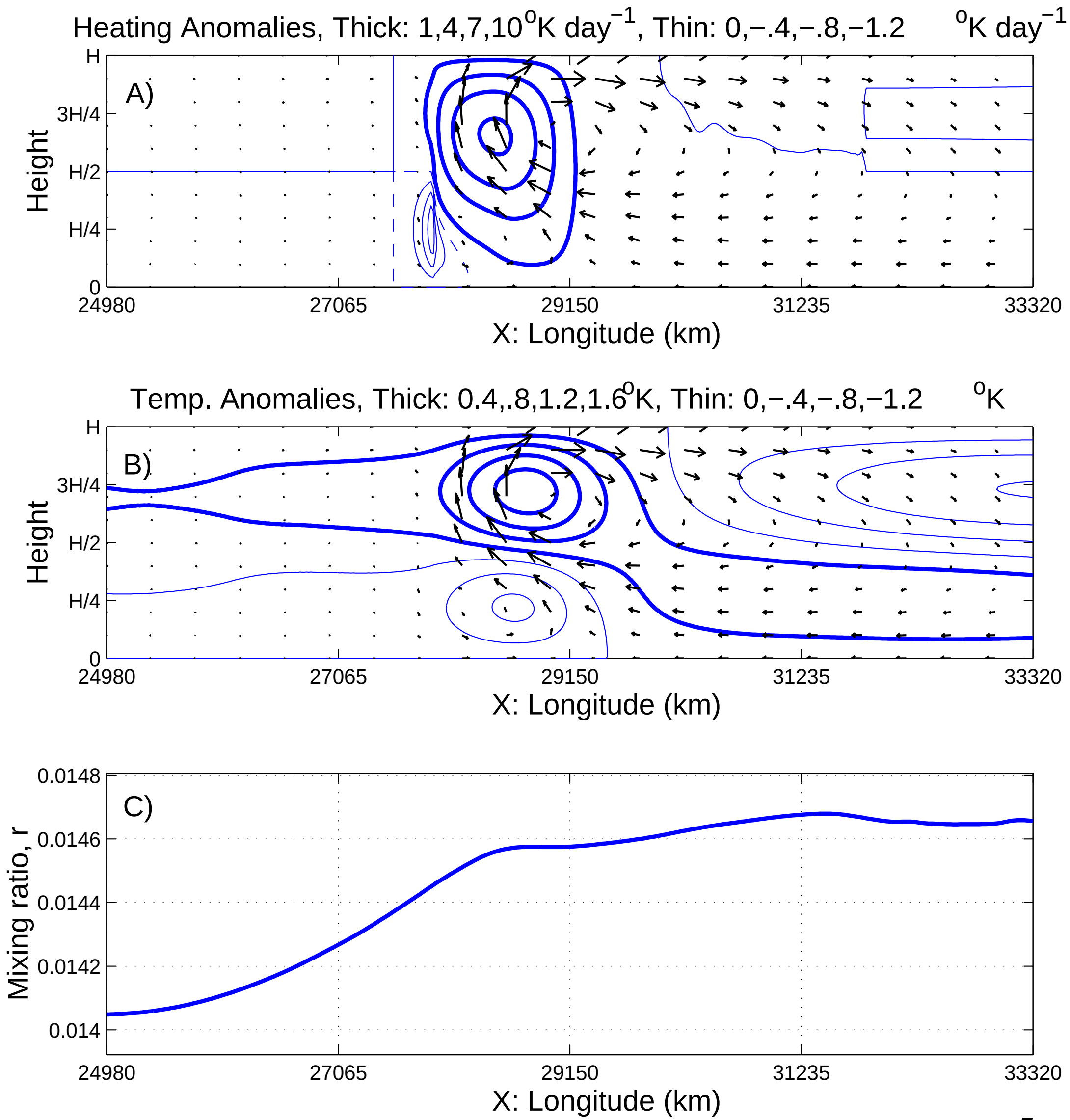
Wave activity

Nonlinear Stratiform waves amplified by WISHE: Eastelies amplify eastward waves and westerlies amplfy westward waves



Vertical structure of nonlinear stratiform waves...

Nevertheless the vertical structure is very encouraging: wave tilt, drying after the passage of wave etc.



Multicloud models

Motivation

- Commonly used convective closures
  - Low-level convergence driven models (CISK): Yamasaki (1969), Hayashi (1971), Lindzen (1974)
  - Quasi-equilibrium: Arakawa-Shubert (1974), surface flux triggers e.g. WISHE (Emanuel 1987)
- **Trimodal nature of tropical convection not explicitly taken into account**.
- Simple models with crude vertical resolution, reduced to one or two baroclinic modes, are useful testing tools
- Stratiform instability, Mapes (2000), Majda-Shefter (2001), Majda et al. (2004)
- Models with three cloud types, K-Majda (2006): deep, stratiform, and congestus, low-level moisture convergence, moisture preconditioning.

Related papers

- K. & Majda, 2006: —– *Part I: Linear analysis.* JAS
- K. & Majda, 2007: —- *Part II: Nonlinear simulations.* JAS
- K. & Majda, 2006: —- *Detailed Nonlinear Wave Evolution.* DAO
- K. & Majda, 2006: *Extensive sensitivity analysis, chaotic regimes.* TCFD
- Majda, K.,& Stechmann, 2007: *MJO analog.* PNAS
- K. & Majda, 2007: *Enhanced congestus closure.* JAS (In press).

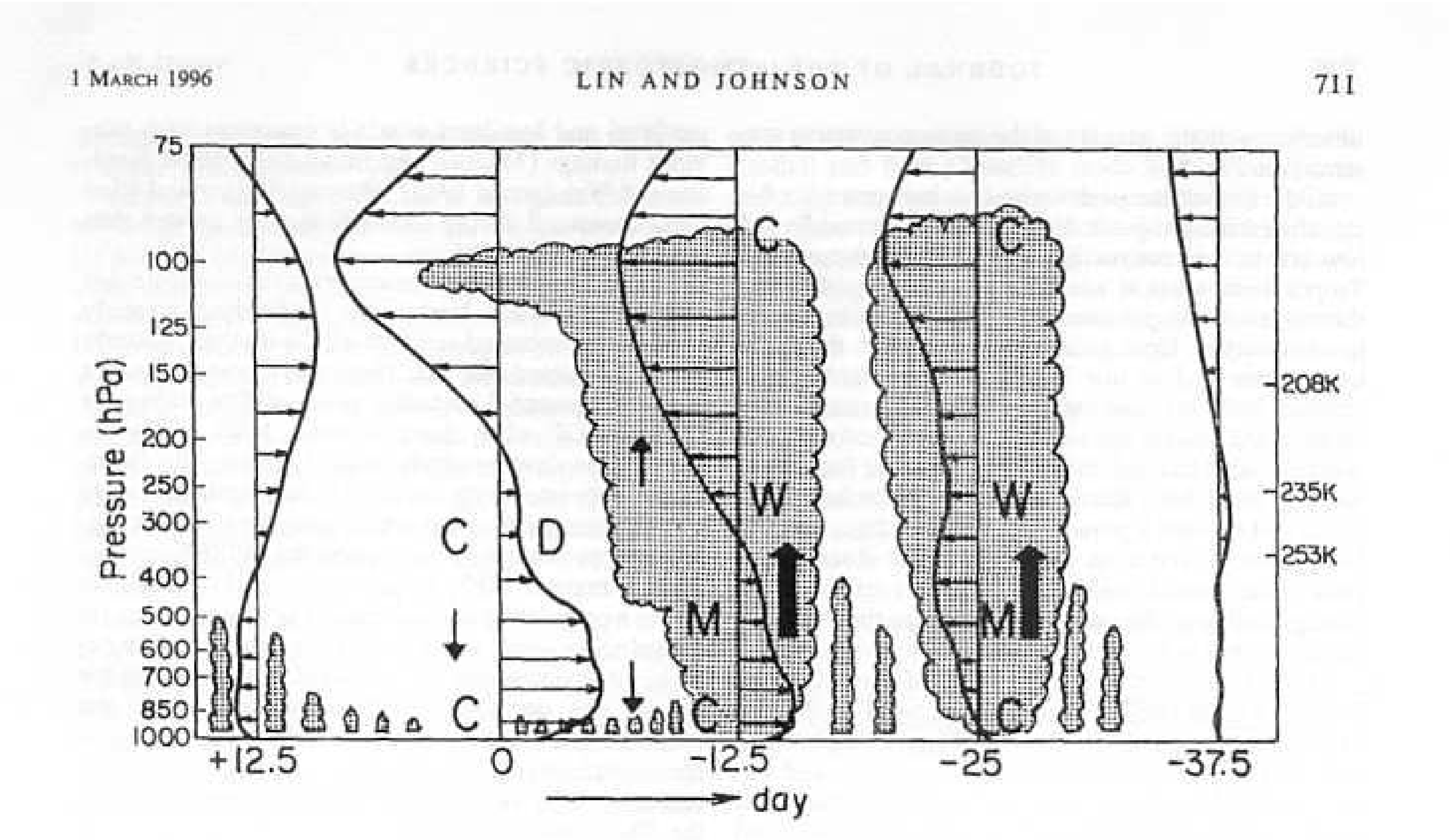
Recall observed features of Kelvin waves

- Propagate eastward at 15-20 m/s, wavelength of about 2000-5000 km.
- Heating region in phase with maximum upward motion and characterized by cold temp. anomalies in lower troposphere and warm aloft.
- Front to rear vertical tilt in wind, temperature, moisture, and heating fields; boomerang shape
- θ_{eb} (CAPE) rises in front and decreases rapidly (CAPE consumed) after passage of wave,
- Trailing stratiform wake.
- Shallow to midlevel convection leads the wave: preconditioning by moistening
- Low level moisture convergence leads the wave

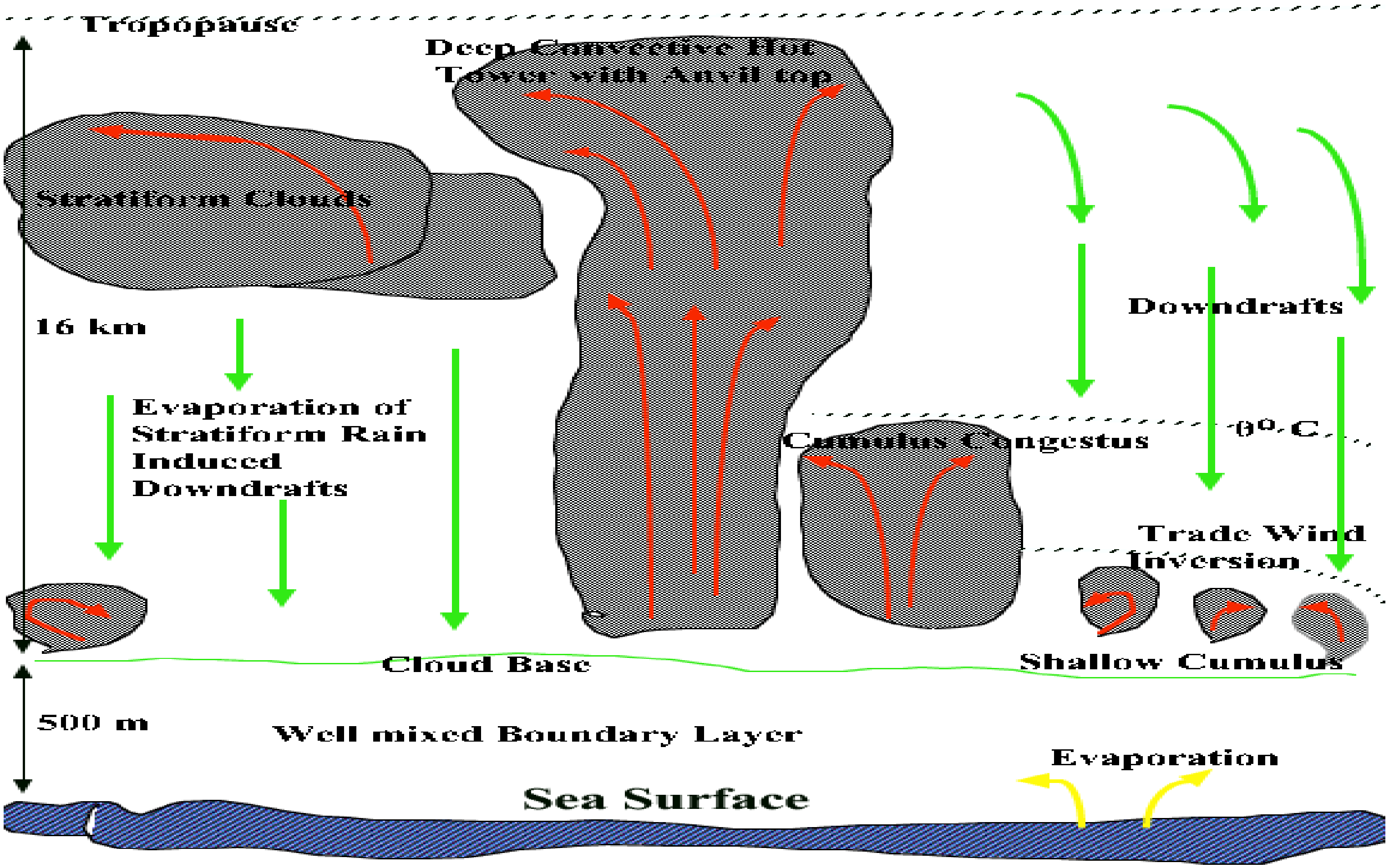
Self-similarity and MJO structure

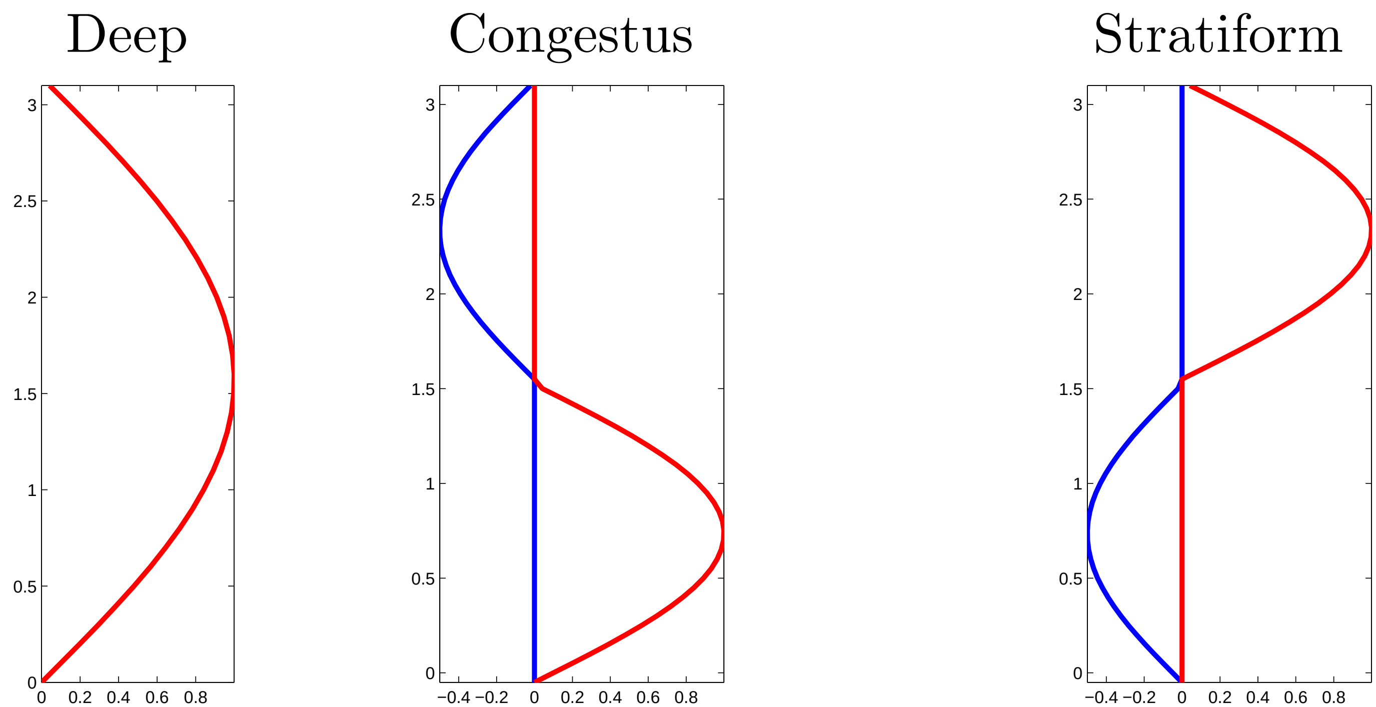
- Most of items in previous list are common features of convectively coupled tropical waves, westward 2 day waves, inertia-gravity waves, mixed-Rossby gravity waves, and the MJO
- MJO has a planetary scale wavelength, wavenumber 1 or 2
- Low frequency mode, periods of 30-60 days, 5-10 m/s
- Envelope of higher frequency mesoscale cloud clusters and/or superclusters; e.g 2 day waves and Kelvin waves
- Stronger westerly wind burst (lasting about 30 days) on west side and expands to the active phase of MJO

Kinematics of tropical convection: A flavor



THE THREE CLOUD MODEL: An idealized picture





$$0 \leq z \leq \pi/2 \qquad H_s(z) = 0;\ \ H_c(z) = H_c \sin(2z)$$

$$E_s(z) = -\delta_s H_s \sin(2z);\ \ E_c(z) = 0$$

$$H_d(z) = H_d \sin(z)$$

$$\pi/2 \leq z \leq \pi \qquad H_s(z) = -H_s \sin(2z);\ \ H_c(z) = 0$$

$$E_s(z) = 0;\ \ E_c(z) = \delta_c H_c \sin(2z)$$

$\delta_{s,c}$: frac. of stratiform/congestus cooling in lower/upper tropo.

Vertical Structure

Galerkin Truncation of Hydrostatic Primitive Equations

- Horizontal Velocity:

$$\mathbf{V} = \bar{\mathbf{U}} + \sqrt{2}\cos\left(\frac{\pi z}{H_T}\right)\mathbf{v}_1 + \sqrt{2}\cos\left(\frac{2\pi z}{H_T}\right)\mathbf{v}_2$$

- Vertical velocity:

$$w = -\frac{H_T}{\pi}\sqrt{2}\left[\sin(\frac{z\pi}{H_T})\ \mathrm{div}\mathbf{v}_1 + \frac{1}{2}\sin(\frac{2\pi z}{H_T})\ \mathrm{div}\mathbf{v}_2\right]$$

- Potential temperature:

$$\Theta = z + \sqrt{2}\sin(\frac{\pi z}{H_T})\ \theta_1 + 2\sqrt{2}\sin(\frac{2\pi z}{H_T})\theta_2.$$

Governing Equations

1st Baroc.
$$\begin{cases} \dfrac{\bar{d}\mathbf{v}_1}{dt} + \beta y \mathbf{v}_1^\perp - \nabla\theta_1 = -C_d(u_0)\mathbf{v}_1 - \dfrac{1}{\tau_R}\mathbf{v}_1 \\ \dfrac{\bar{d}\theta_1}{dt} - \operatorname{div}\mathbf{v}_1 = H_d + \xi_s H_s + \xi_c H_c + S_1 \end{cases}$$

2nd Baroc.
$$\begin{cases} \dfrac{\bar{d}\mathbf{v}_2}{dt} + \beta y \mathbf{v}_2^\perp - \nabla\theta_2 = -C_d(u_0)\mathbf{v}_2 - \dfrac{1}{\tau_R}\mathbf{v}_2 \\ \dfrac{\bar{d}\theta_2}{dt} - \dfrac{1}{4}\operatorname{div}\mathbf{v}_2 = (-H_s + H_c) + S_2. \end{cases}$$

Moist thermodynamics

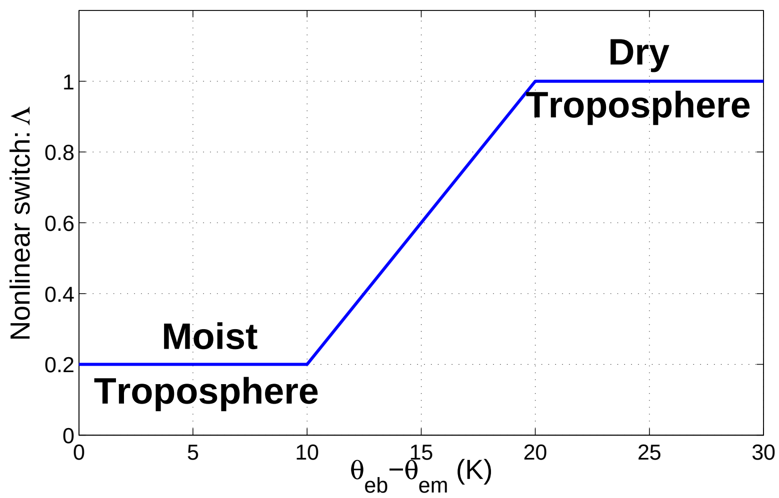
- Moisture Eqn:
$$\frac{\bar{d}q}{dt} + \mathrm{div}\left[(\mathbf{v}_1 + \tilde{\alpha}\mathbf{v}_2)q + \tilde{Q}(\mathbf{v}_1 + \tilde{\lambda}\mathbf{v}_2)\right] = -P + \frac{D}{H_T}$$
$$P = \frac{2\sqrt{2}}{\pi}(H_d + \xi_s H_s + \xi_c H_c)$$
ξ_s, ξ_c : add contribution from congestus and stratiform clouds to surface precip.

- Boundary layer: $$\frac{\partial \theta_{eb}}{\partial t} = \frac{1}{h_b}(E - D)$$

- **Constraint**: Conservation of moist static energy
$$\frac{\partial \langle \theta_e \rangle_z}{\partial t} = \frac{1}{H_T}E + \frac{2\sqrt{2}}{\pi}S_1$$
with $\langle \theta_e \rangle_z = \frac{h_b}{H_T}\theta_{eb} + \frac{1}{H_T}\frac{2\sqrt{2}}{\pi}\theta_1 + q$

Convective Parametrization: Moisture Trigger

- Middle tropospheric $theta_e$:
  $\theta_{em} = q + \frac{2\sqrt{2}}{\pi}(\theta_1 + \alpha_2\theta_2)$
- Switch function:
  $\Lambda^* \leq \Lambda \leq 1$



- Preferential convective heating profile

$$\text{Deep: } H_d = \frac{1-\Lambda}{1-\Lambda^*} Q_d,$$

$$\text{Congestus: } \frac{\partial H_c}{\partial t} = \frac{1}{\tau_c}\left(\alpha_c \frac{\Lambda - \Lambda^*}{1-\Lambda^*} Q_c - H_c\right)$$

$$\text{Stratiform: } \frac{\partial H_s}{\partial t} = \frac{1}{\tau_s}(\alpha_s H_d - H_s)$$

Convective closures

- Quasi-equilibrium: CAPE and Betts-Miller like scheme

$$Q_d = \frac{1}{\tau_{conv}} \left[a_1 \theta_{eb} + a_2 (q - \hat{q}) - a_0 (\theta_1 + \gamma_2 \theta_2) \right]^+$$

$$Q_c = \left[\theta_{eb} - a_0' (\theta_1 + \gamma_2 \theta_2) \right]^+ \text{ or } Q_c = \frac{D}{H}$$

- Downward Mass-Flux

$$D_0 = \frac{m_0}{Q_{R,1}^0} \left[\bar{Q}_{R,1}^0 + \mu (H_s - H_c) \right]^+ (\theta_{eb} - \theta_{em})$$

$$D = D_0 \text{ or } D = \Lambda D$$

- Newtonian radiative cooling & Sea surface evaporation

$$S_j = -Q_{R,j}^0 - \frac{1}{\tau_D} \theta_j, \qquad j = 1, 2; \quad \frac{1}{h} E = \frac{1}{\tau_e} (\theta_{eb}^* - \theta_{eb})$$

Some characteristic features of multicloud models

- Vertical integral of moist static energy is conserved
- Moisture switch, Λ, inhibits deep convection and favours congestus clouds in dry atmosphere regions
- Congestus preconditioning prior to deep convection through low-level moisture convergence

$$\frac{\partial \theta_2}{\partial t} - \frac{1}{4}\mathrm{div}\mathbf{v}_2 \approx H_c > 0 \implies -\mathrm{div}\mathbf{v}_2 > 0 \implies \frac{\partial q}{\partial t} \approx -\tilde{\lambda}\mathrm{div}\mathbf{v}_2 > 0$$

- Convective closures combines CAPE and Betts-Miller adjustment concepts.
- Purely thermodynamic boundary layer with prescribed SST supplies evaporative forcing with no WISHE effect.
- Stratiform heating with a few hours lag and downdrafts cool and dry the boundary layer and moisten the middle troposphere
- Uniform radiative cooling

Radiative convective equilibrium and linear waves

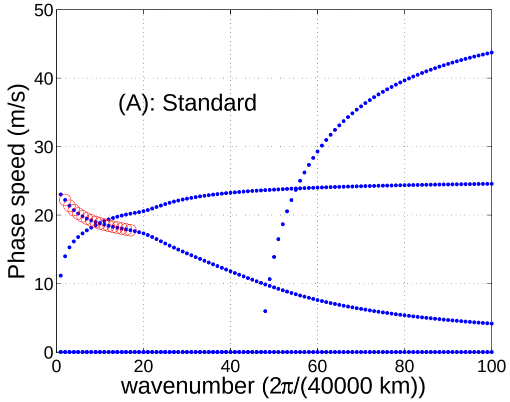
- A RCE is a time independent and homogeneous solution

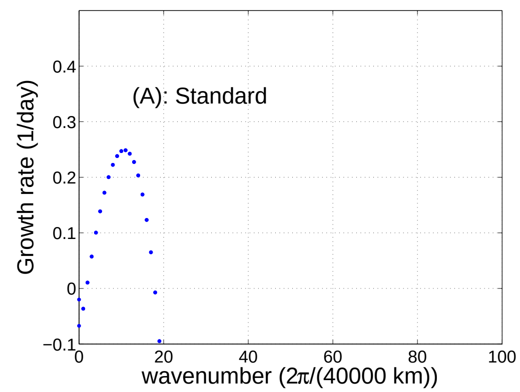
  Radiative cooling $\equiv$ convective heating $\equiv Evaporative forcing$

- Uniquely determined if $Q^0_{R,1}, \theta^*_{eb} - \bar{\theta}_{eb}, \bar{\theta}_{eb} - \bar{\theta}_{em}$ are given.
- Basic state about which convectively coupled waves propagate and grow.
- Linearized system: $U_t + AU_x = BU$
- Seek linear wave solution: $U(x,t) = U \exp(i(kx - \omega t))$
- $\mathrm{Re}(\omega)/k$: phase speed, $\mathrm{Imag}(\omega)$: growth

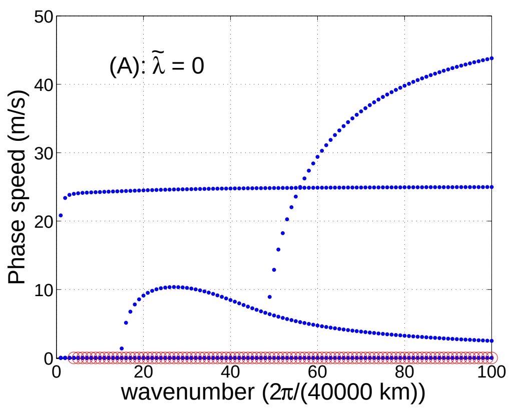
Linear Theory: Basic Moist Gravity Wave Instability

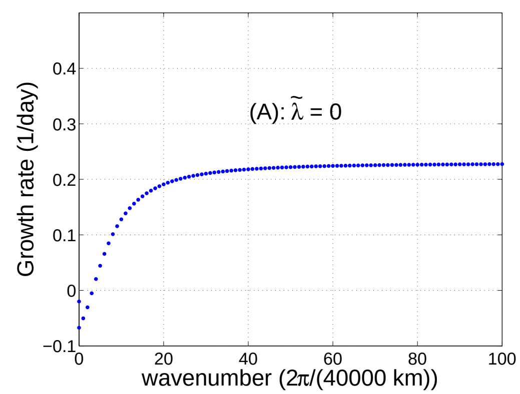
A) $\tilde{\lambda} = 0.8, \bar{\theta}_{eb} - \bar{\theta}_{em} = 14, \gamma_2 = .1, \mu_2 = .5, a_0 = 7.5, a_1 = .1, a_2 = .9$ K





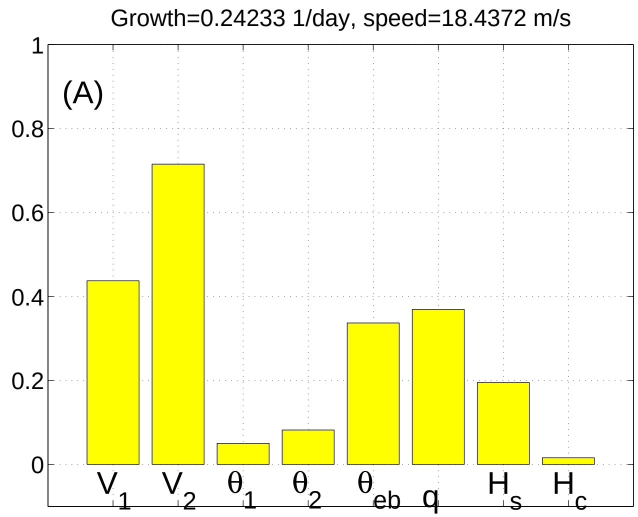
B) Ignoring 2nd Baroc. moisture Convergence

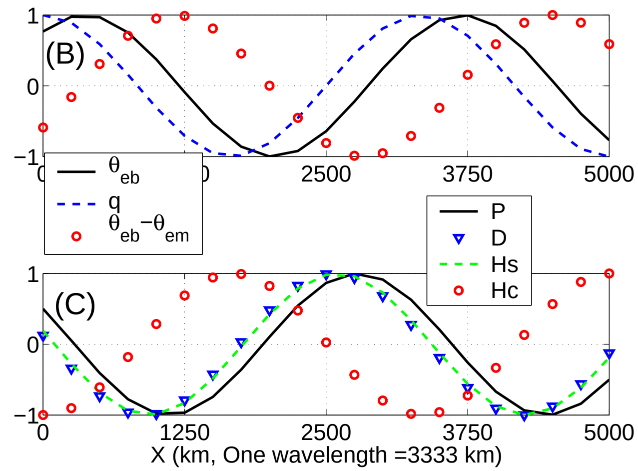


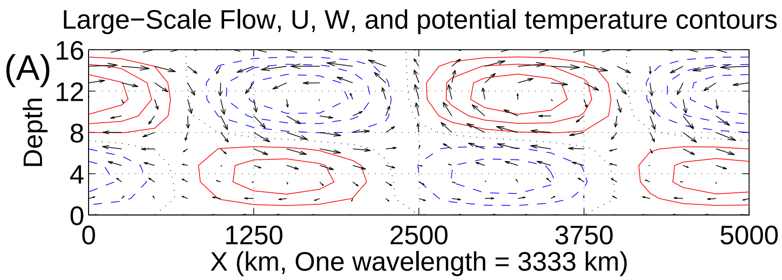


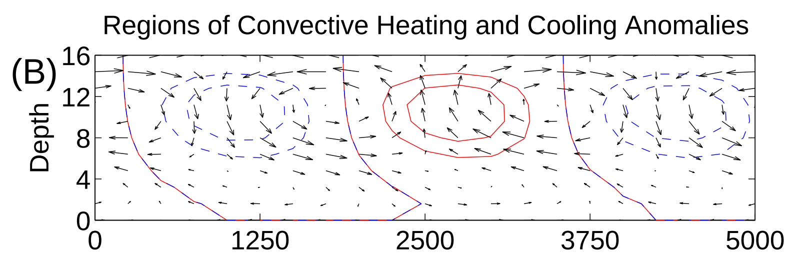
('Radiative instability' in one vertical mode models)

Moist gravity waves: Physical structure and dynamical features





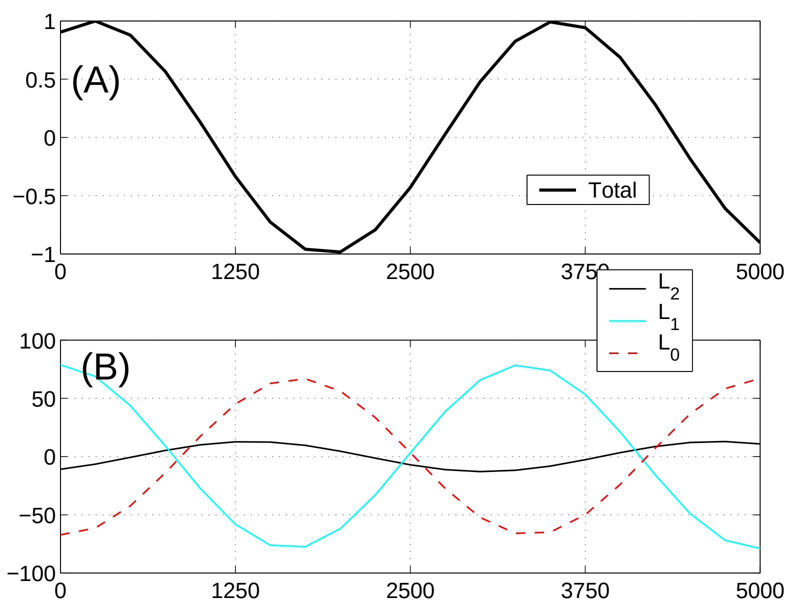




Observed tropical superclusters.

Deep Convective Budget

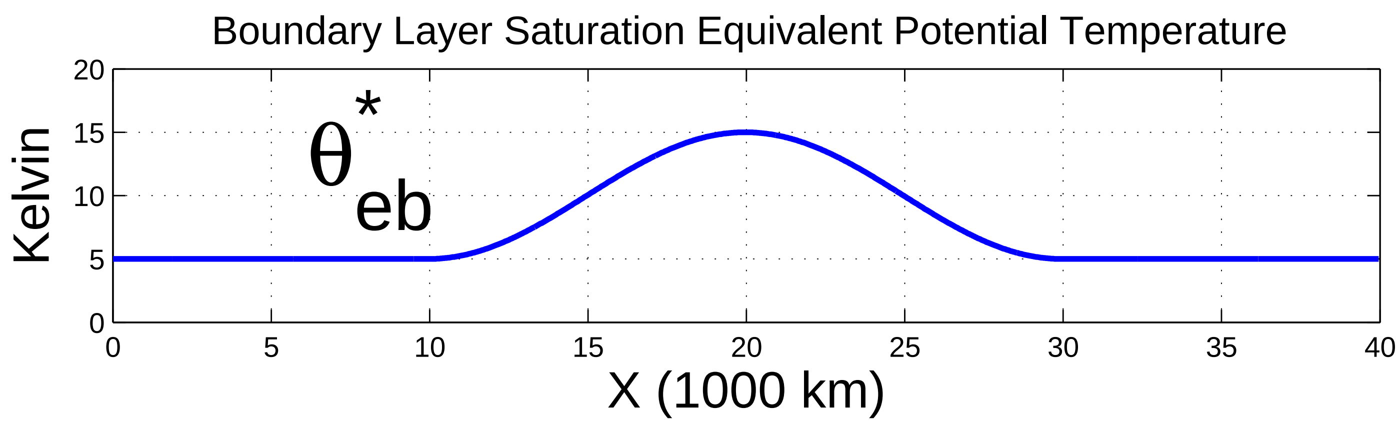
$$\frac{\partial P}{\partial t} = L_2(\theta_2, H_s, u_2, H_c) + L_1(\theta_1, u_1) + L_0(q, \theta_{eb}).$$



- Moist thermodynamics pre-conditioning
- Second baroclinic birth stage (low level moisture convergence; congestus heating in lower troposphere)
- First baroclinic amplification/deep convective stage

Nonlinear Simulations: Setup

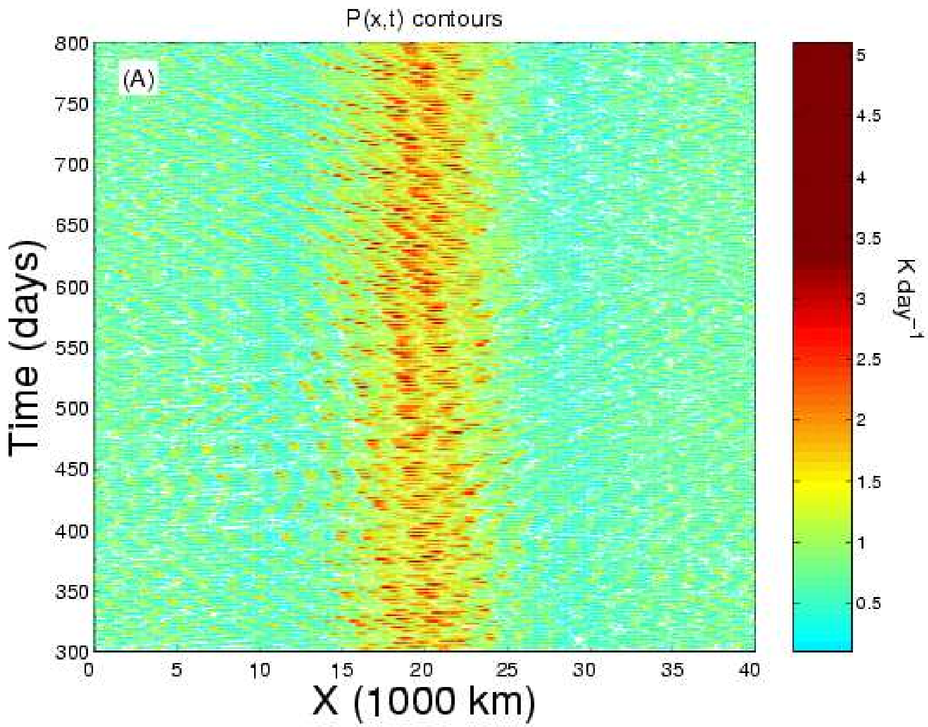
- Periodic domain of 40,000 km, no rotation
- $\Delta x = 40$ km, CFL based Time step. (used 2 minutes)
- Walker circulation: imposed region of enhanced SST (Indian Ocean/Western Pacific warm pool)



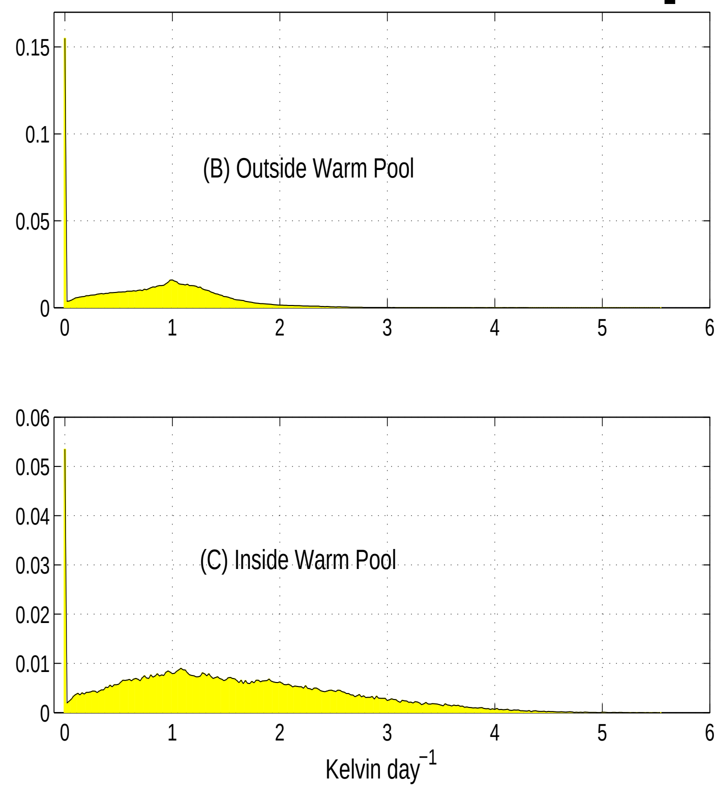
- Integrate to statistical steady state

Convective activity

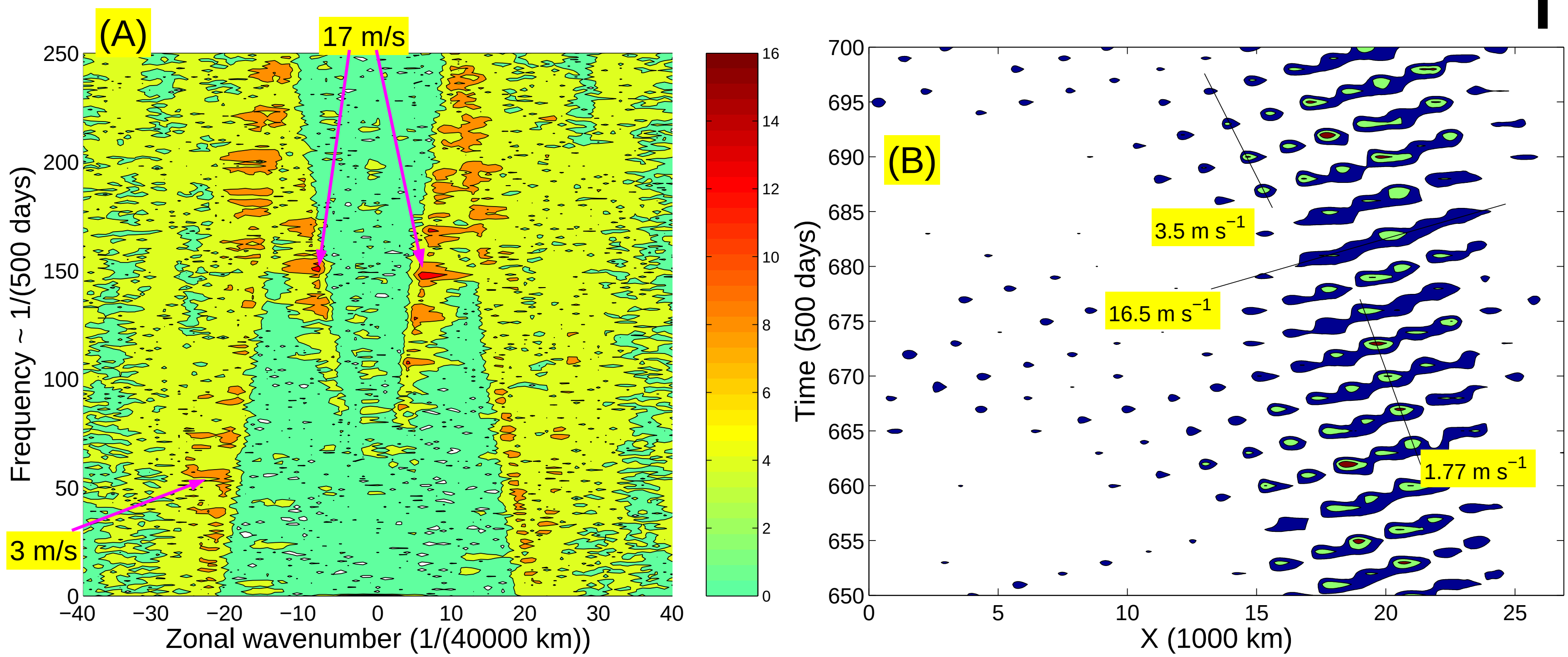
P(x,t) contours



PDF's



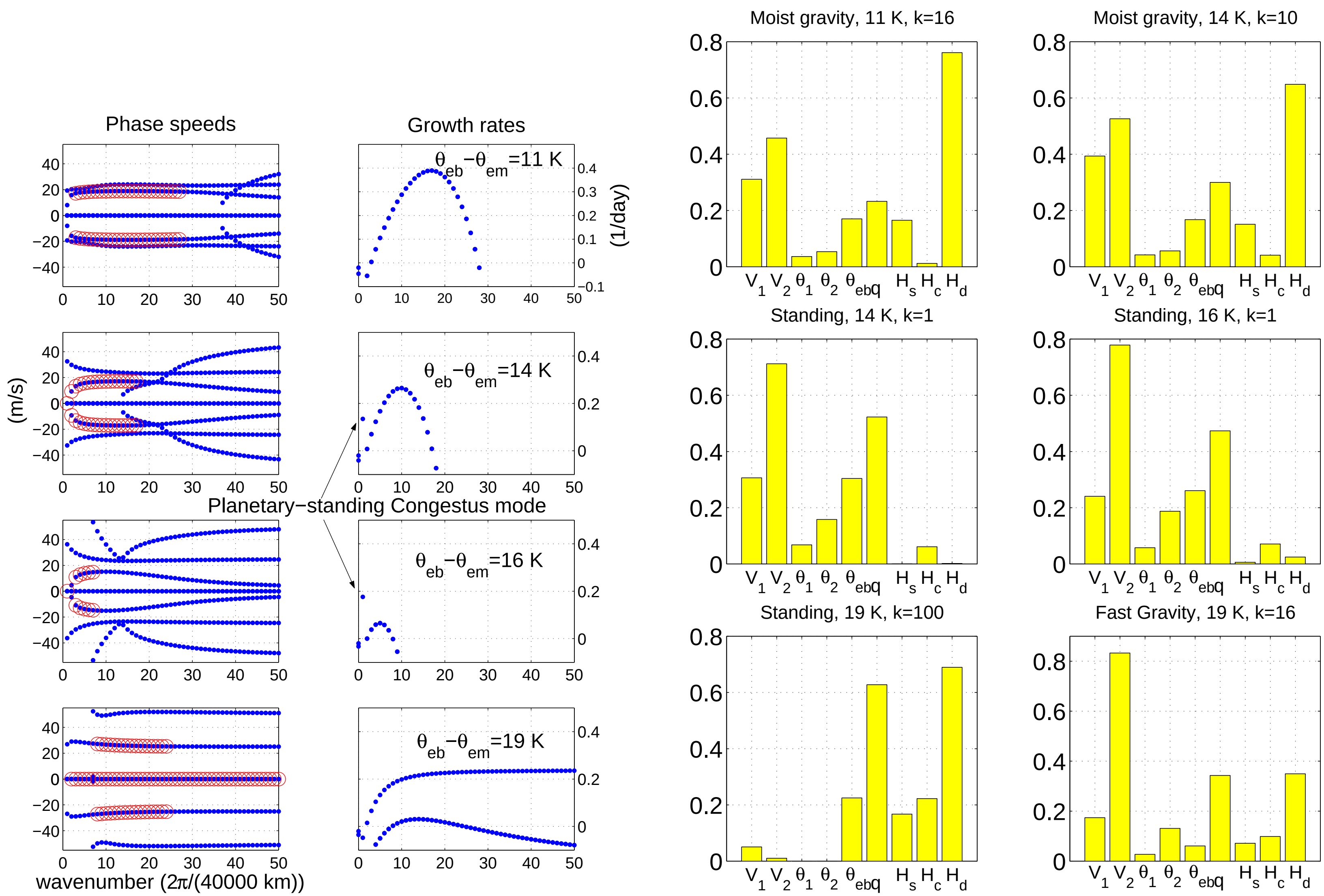
Slowly propagating westward envelopes of moist gravity waves (Precipitation/deep convection)



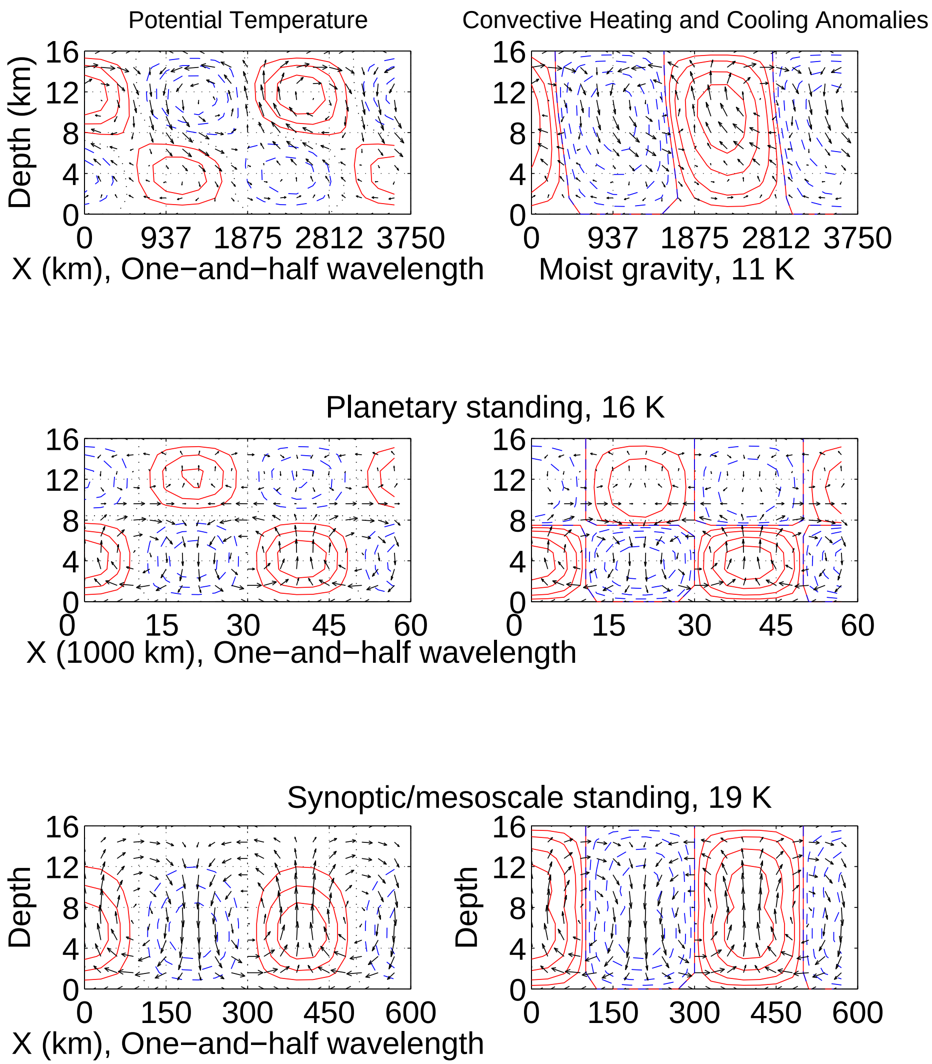
Linear and nonlinear waves in the enhanced congestus multicloud model

$$Q_c = \frac{1}{\tau_{conv}}(a_1\theta_{eb} + a_2 q - a_0(\theta_1 + \gamma_2\theta_2));$$

$$D = D_0$$


Phase speeds
Growth rates
(m/s)
(1/day)
wavenumber (2π/(40000 km))
θeb−θem=11 K
θeb−θem=14 K
θeb−θem=16 K
θeb−θem=19 K
Planetary−standing Congestus mode
Moist gravity, 11 K, k=16
Moist gravity, 14 K, k=10
Standing, 14 K, k=1
Standing, 16 K, k=1
Standing, 19 K, k=100
Fast Gravity, 19 K, k=16
V1 V2 θ1 θ2 θebq Hs Hc Hd

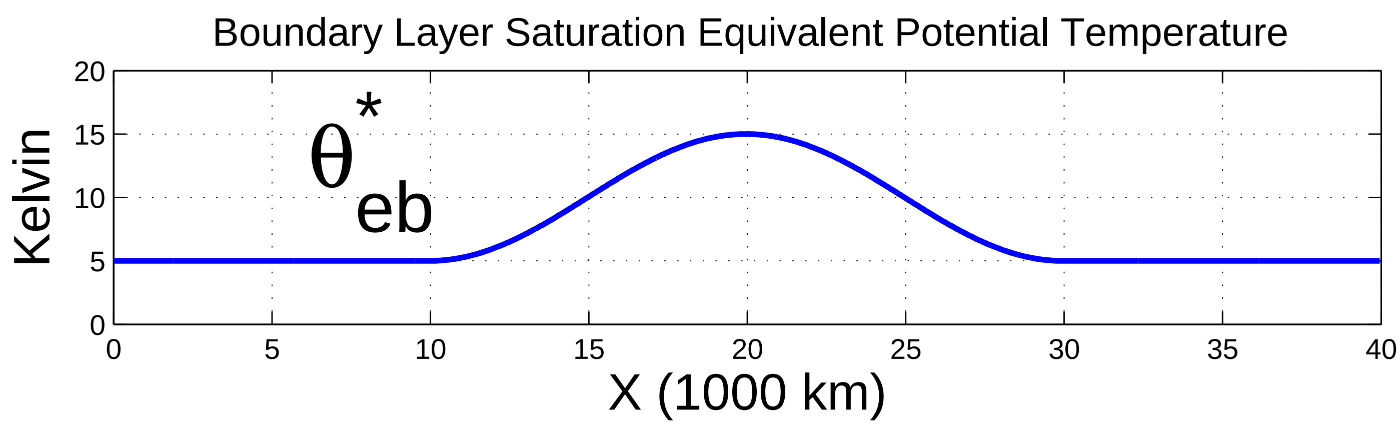
Physical structure of unstable waves



Nonlinear Simulations: Setup

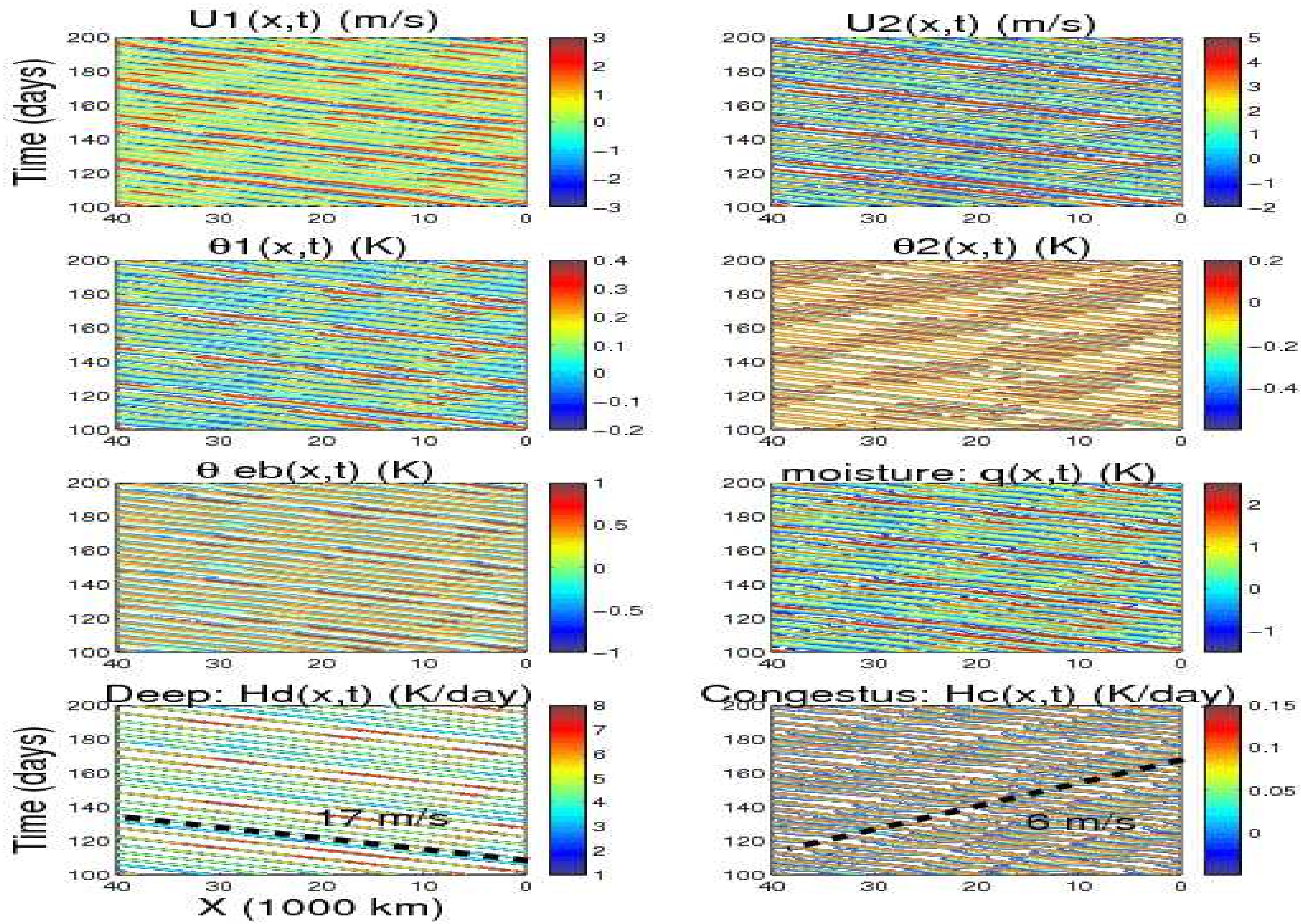
- Nonlinearities: moisture convergence ($\mathrm{div}\mathbf{v}_1 + \tilde{\alpha}\mathbf{v}_2$), convective switches.
- Periodic domain of 40,000 km, no rotation
- Three regimes:

1) Aquaplanet with uniform SST
2) Imposed SST gradient mimicking Indian Ocean/Western Pacific warm pool

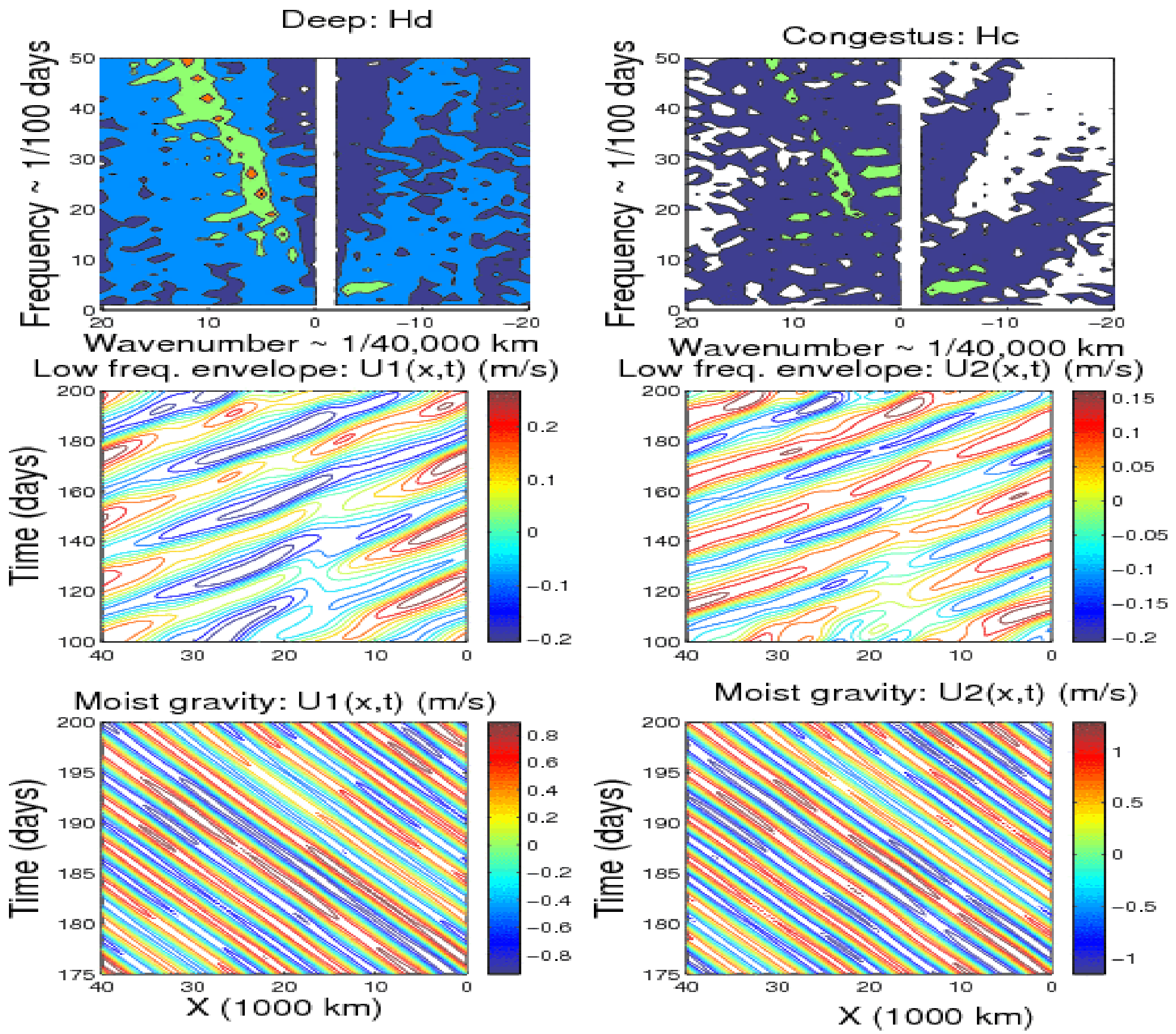


3) Planetary congestus mode driven regime ($\bar{\theta}_{eb} - \bar{\theta}_{em} = 16$ K)

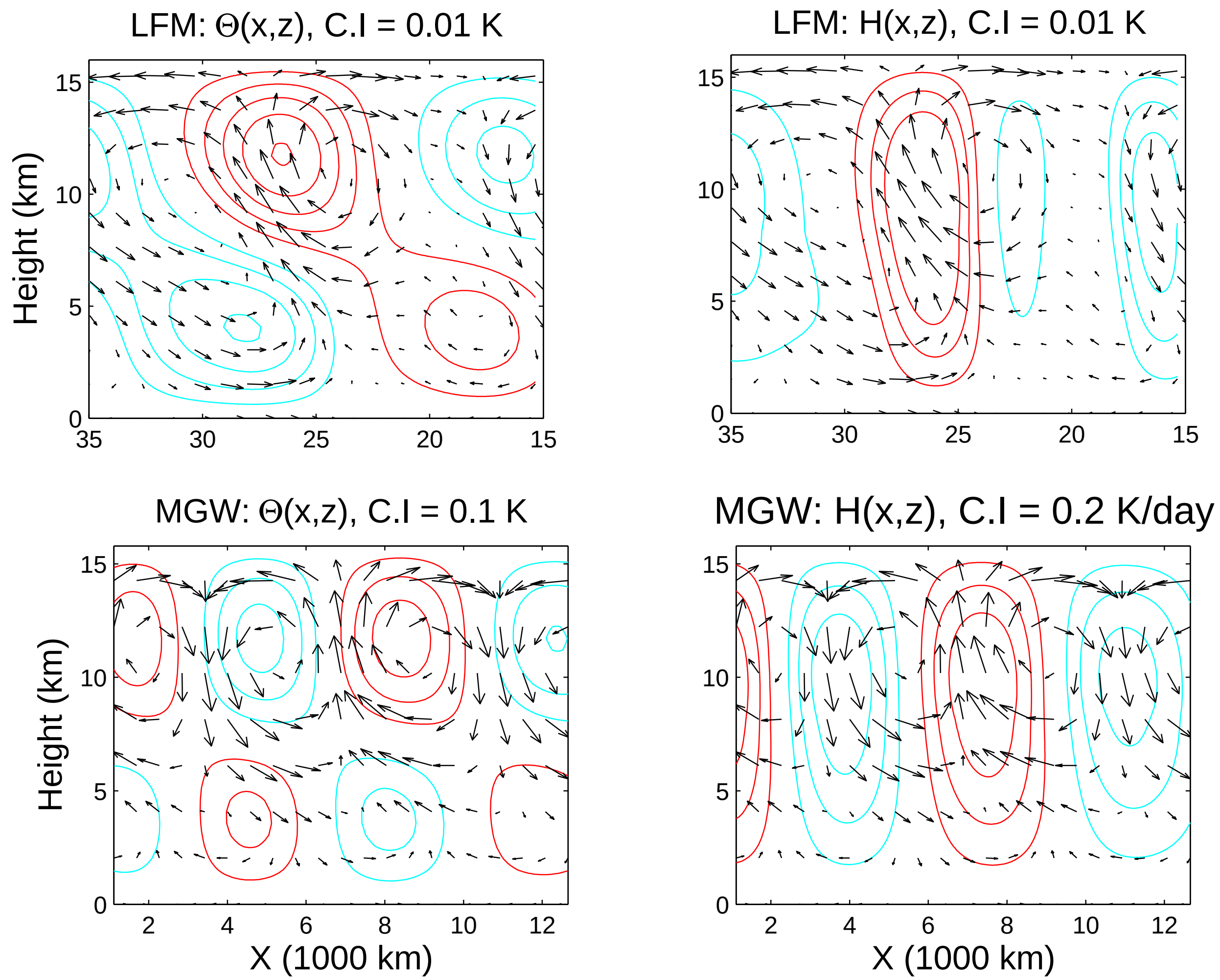
Uniform SST: MJO-like wave envelopes



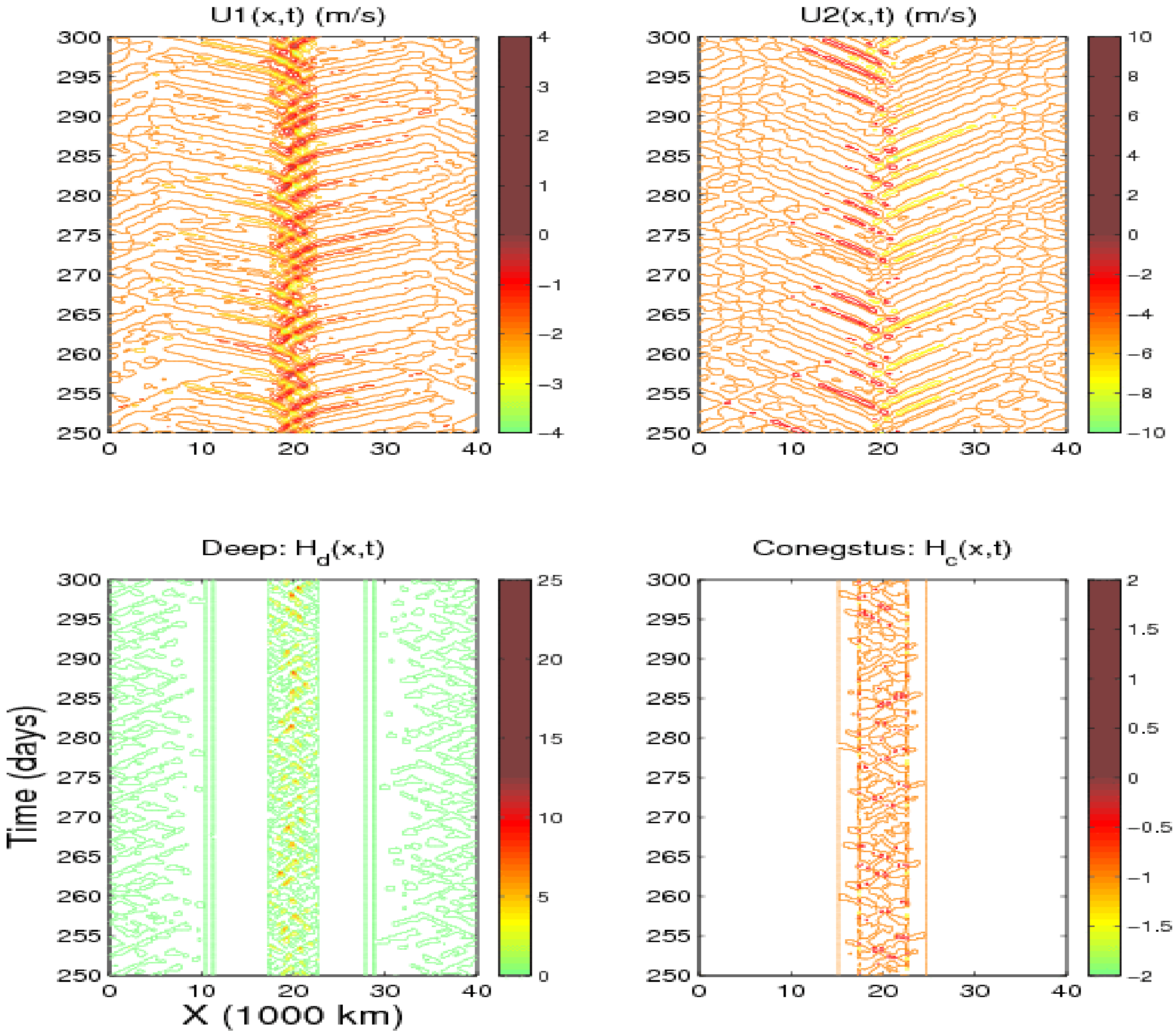
Low frequency envelopes / Moist gravity waves

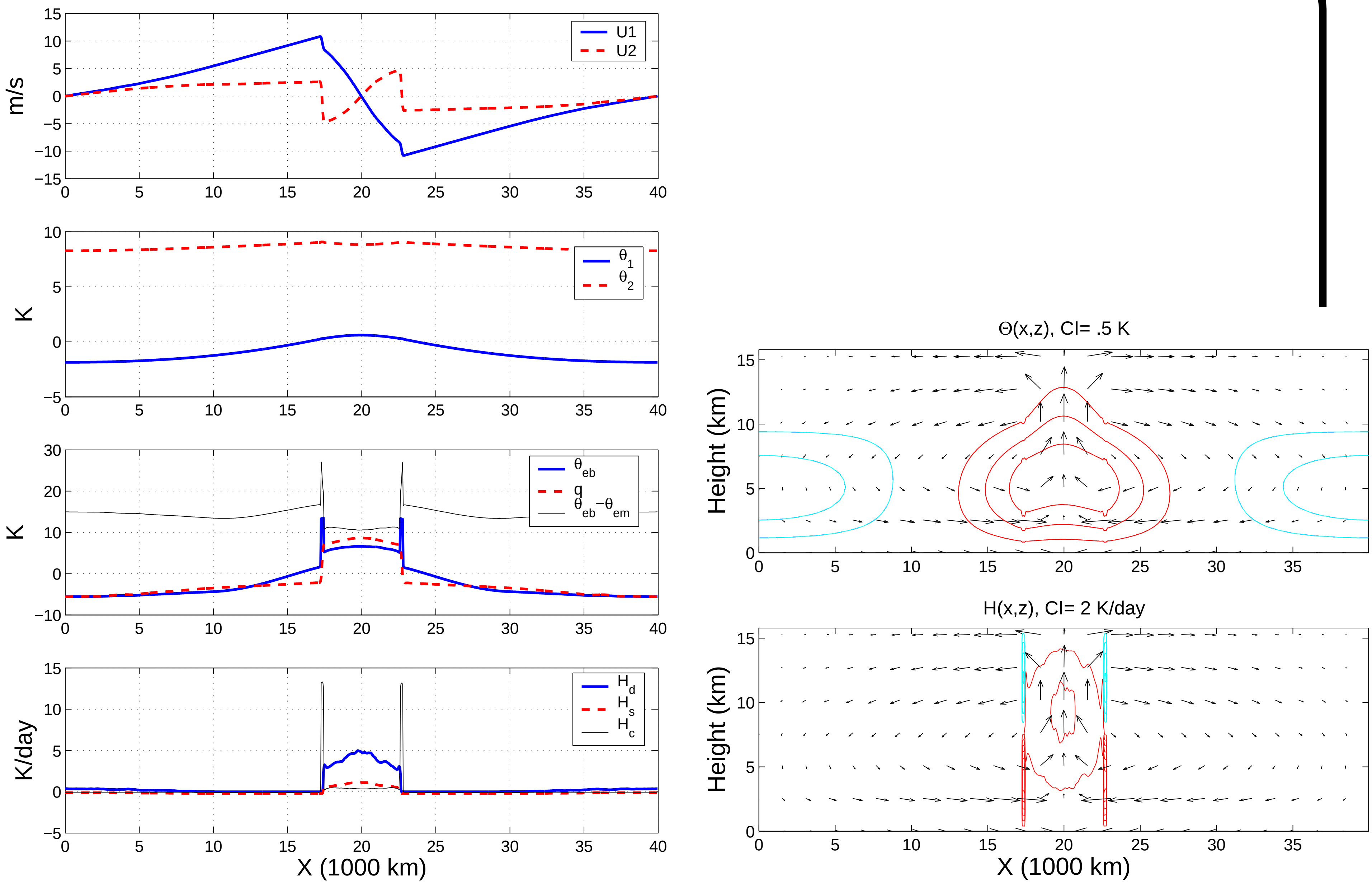


Self-similar structure: MJO/Kelvin wave-like



Warm-pool SST: Congestus barriers

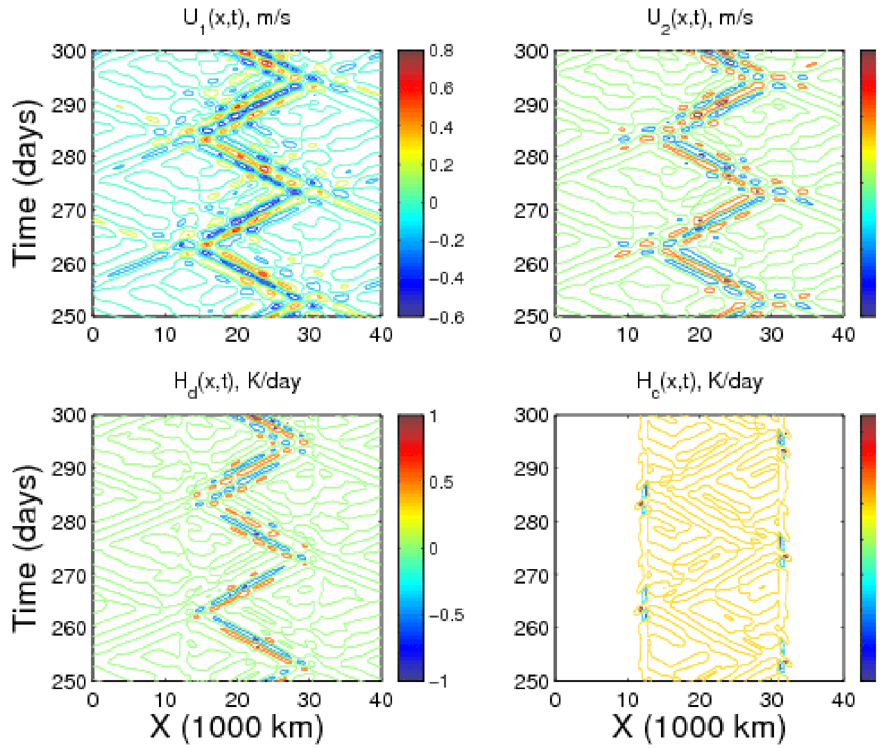


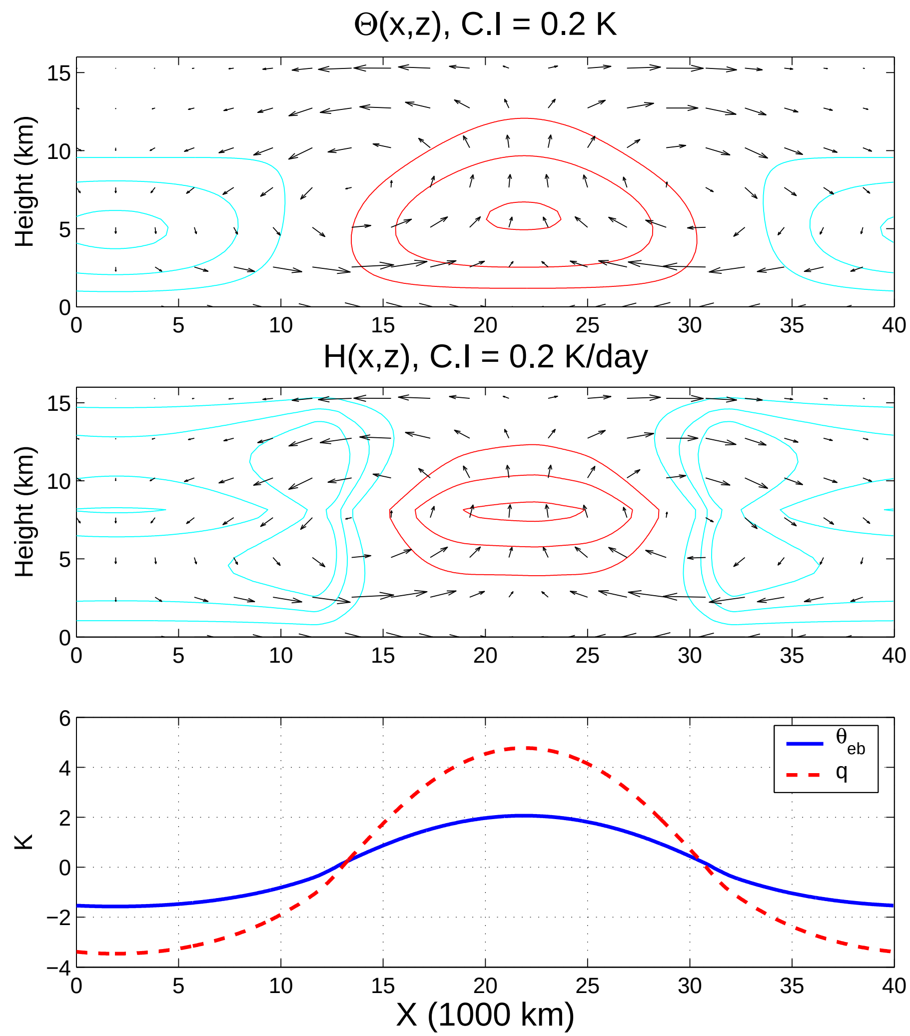


U1
U2
m/s
θ1
θ2
K
θeb
q
θeb−θem
K
Hd
Hs
Hc
K/day
X (1000 km)
Θ(x,z), CI= .5 K
Height (km)
H(x,z), CI= 2 K/day
Height (km)
X (1000 km)

Interaction with planetary congestus mode:

$\bar{\theta}_{eb} - \bar{\theta}_{em} = 16$ K





Summary of Multicloud model features:

- Three cloud types: deep convective, stratiform & congestus.
- Moisture equation with both first and second baroclinic moisture convergence (No CISK). Coupled thermodynamically active boundary layer (No WISHE).
- Moisture switch function inhibits deep convective and favours congestus clouds in dry regions
- Congestus cloud decks in front and induced low-level convergence precondition and moisten the lower troposphere prior to deep convection.
- Stratiform rain trails the wave. It cools and dries the boundary layer through downdrafts.
- Scale selective instability without CISK, WISHE, beta, or cloud radiative feedback
- Low level congestus heating in front and upper level stratiform heating in the wake implies a tilt in the heating field...

Multicloud model summary: Key results

- Synoptic scale (linear) moist gravity wave packets moving at 15-20 m/s with (nonlinear) low frequency planetary scale envelopes (LFE) moving in opposite direction at about 3-6 m/s. Self-similar structure resembling convectively coupled waves, reminiscent of the MJO.
- Large scale convective cycle: CAPE generation, congestus moistening, deep convective birth and amplification, then demise of wave (K.& Majda, DAO, 2006)
- Congestus heating leads and is active during preconditioning and moistening phase
- Low-level moisture convergence plays a central role during preconditioning stage (2nd baroclinic convergence)
- Planetary scale congestus mode creates a moist region where moist gravity waves develop and propagate. This results in a Walker circulation with uniform SST.