1 The Nash-Moser method and applications

Outline of the Course by Massimiliano Berti and Philippe Bolle:

- Lecture 1: Periodic and quasiperiodic solutions near an elliptic equilibrium for Hamiltonian PDEs: presentation of the problem. We shall specially focus on periodic solutions of nonlinear wave equations.

  Lyapunov-Schmidt reduction: the range and the bifurcation equations.

  Small denominator problem and statement of a Nash Moser implicit function theorem for the range equation.

  Variational structure of the bifurcation equation.

- Lectures 2: Nash Moser-type iterative scheme. Convergence proof, under appropriate weak invertibility assumptions on the linearized problems.

- Lectures 3-4: Inversion of the linearized equations in presence of small divisors for periodic solutions in any spatial dimensions.

References

[1] M. Berti, P. Bolle, *Cantor families of periodic solutions for completely resonant nonlinear wave equations*, Duke Math. J. 134 (2006) 359-419.

[2] M. Berti, P. Bolle, *Cantor families of periodic solutions for wave equations via a variational principle*, Advances in Mathematics. 217 (2008) 1671-1727.

[3] M. Berti, P. Bolle, *Sobolev periodic solutions of nonlinear wave equations in higher spatial dimensions*, preprint 2008.

[4] J. Bourgain , *Green's function estimates for lattice Schrödinger operators and applications*, Annals of Mathematics Studies 158, Princeton University Press, Princeton, 2005.

[5] W. Craig, *Problèmes de petits diviseurs dans les équations aux dérivées partielles*, Panoramas et Synthèses, 9, Société Mathématique de France, Paris, 2000.

[6] S. Kuksin, *Analysis of Hamiltonian PDEs*, Oxford Lecture series in Mathematics and its applications 19, Oxford University Press, 2000.