Logarithmic Sobolev inequality for spin systems

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Abstract

In the first part of the talk, we present a criterion for the logarithmic Sobolev inequality (LSI) on the product space $X_1 \times \ldots \times X_N$. We have in mind an N-site lattice, unbounded continuous spin variables, and Glauber dynamics. The interactions are described by the Hamiltonian H of the Gibbs measure. The criterion for LSI is formulated in terms of the LSI constants of the single-site conditional measures and the size of the off-diagonal entries of the Hessian of H. It is optimal for Gaussians with positive covariance matrix. To illustrate, we give two applications: one with weak interactions and one with strong interactions and a decay of correlations condition. This is joint work with Maria Reznikoff.

In the second part of the talk, we present a similar abstract framework for a hydrodynamic limit. It is formulated in terms of a projection $P: X \to Y$ on coarse grained variables. We apply this framework to rederive a hydrodamic limit for a conservative spin system with Kawasaki dynamics. This is joint work with Natalie Grunewald, Maria Reznikoff and Cedric Villani.