The Infinity Laplacian: from classical analysis to image processing and random turn games

Adam Oberman
Simon Fraser University

Abstract

The Infinity Laplacian equation is currently at the interface of a number of different mathematical fields. It was first studied in the 1950s by the Swedish mathematician Gunnar Aronsson, motivated by classical analysis problem of building Lipschitz extensions of a given function. While Aronsson was able to find interesting exact solutions, progress stalled because solutions were non-classical. It took another forty years until analytical tools were developed to study the equation rigorously, and computational tools were developed which made numerical solution of the equation possible.

In the last decade, PDE theorists established existence and uniqueness, and (quite recently) appropriate regularity results. At the same time, the image processing community was using the operator for edge detection, and for inpainting, the reparation of images with damage. While the operator was promising, they had little success, since traditional methods for solving the equation yielded poor results.

It turns out that the right way to solve the equation is to go back to the original Lipschitz extension problem. This leads to a formula for the discrete operator with a simple interpreation, and good solution properties. This formula also leads to another surprising connection with probability theory.

Working in the unrelated field of percolation theory, a group of probalists (Peres-Shramm-Sheffield-Wilson) studying a randomized version of a marble game called Hex found a connection with the Infinity Laplacian equation. This connection gives an interpretation of the equation as a two player random game.

I'll tell this story, and explain some of the more accessible properties of the equation, along with pictures and numerical results.