Pacific Northwest PDE Meeting

2007-2008

Saturday, September 29th 2007

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6th Pacific Northwest PDE Meeting
Simon Fraser University, Burnaby
Saturday, September 29, 2007

Schedule

9:30 a.m.	Coffee and snacks
10:00 a.m. to 10:50 a.m.	Felix Otto (University of Bonn) **Logarithmic Sobolev inequality for spin systems**
10:50 a.m. to 11:40 a.m.	Tatiana Toro (University of Washington) **Boundary structure and size interms of interior and exterior harmonic measure**
11:40 a.m.	Break
11:50 a.m. to 12:40 p.m.	Reinhard Illner (University of Victoria) **Recent Progress on Fokker-Planck Multilane Traffic Flow Models**
12:40 p.m.	Lunch
2:00 p.m. to 2:50 p.m.	Nassif Ghoussoub (University of British Columbia) **Bessel potentials and optimal Hardy and Hardy-Rellich inequalities**
2:50 p.m. to 3:40 p.m.	Adam Oberman (Simon Fraser University) **The Infinity Laplacian: from classical analysis to image processing and random turn games**
3:40 p.m.	Coffee and cake break
4:10 p.m. to 5:00 p.m.	Stephen Gustafson (University of British Columbia) **Landau-Lifshitz dynamics and Schroedinger maps**

All talks will be held in SFU PIMS Site — TASC 2 Building Room 8500

Logarithmic Sobolev inequality for spin systems

Felix Otto
Institute for Applied Mathematics, University of Bonn
10:00 a.m. to 10:50 a.m.

In the first part of the talk, we present a criterion for the logarithmic Sobolev inequality (LSI) on the product space $X_1 \times \ldots \times X_N$. We have in mind an N–site lattice, unbounded continuous spin variables, and Glauber dynamics. The interactions are described by the Hamiltonian H of the Gibbs measure. The criterion for LSI is formulated in terms of the LSI constants of the single–site conditional measures and the size of the off–diagonal entries of the Hessian of H. It is optimal for Gaussians with positive covariance matrix. To illustrate, we give two applications: one with weak interactions and one with strong interactions and a decay of correlations condition. This is joint work with Maria Reznikoff.

In the second part of the talk, we present a similar abstract framework for a hydrodynamic limit. It is formulated in terms of a projection $P: X \to Y$ on coarse grained variables. We apply this framework to rederive a hydrodamic limit for a conservative spin system with Kawasaki dynamics. This is joint work with Natalie Grunewald, Maria Reznikoff and Cedric Villani.

Boundary structure and size in terms of interior and exterior harmonic measure

Tatiana Toro
University of Washington
10:50 a.m. to 11:40 a.m.

In the late 80's Wolff constructed domains in $\mathbf{R^3}$ whose harmonic measure had the Hausdorff dimension either strictly less than 2 or strictly greater than 2. Lewis, Verchota and Vogel [LVV] improved this construction to obtain domains in $\mathbf{R^n}$ whose interior and exterior harmonic measures both have Hausdorff dimension strictly less than $n-1$ or strictly greater than $n-1$. In recent work with C. Kenig and D. Preiss we study the case when the exterior and interior harmonic measures are mutually absolutely continuous. Our results bear some resemblance to McMillan's theorem in $\mathbf{R^2}$.

Recent Progress on Fokker-Planck Multilane Traffic Flow Models

Reinhard Illner
University of Victoria

11:50 a.m. to 12:40 p.m.

I will begin with a brief review of traffic models. The bulk of the lecture will focus on recent mathematical results obtained in collaboration with R. Pinnau and C. Kirchner from the University of Kaiserslautern:

1. It will be shown how the second-order conservation law model due to Aw and Rascle can be derived from a kinetic model of Fokker-Planck type.

2. We present a criterion on braking/acceleration force terms, and on the diffusivity such that the fundamental diagram is computable and one-valued. (i.e., for each density $\rho > 0$ there is a unique average speed u and an equilibrium solution $f(v)$ of the model such that $\int f(v)\ dv = \rho$ and $\int v f(v)\ dv = \rho u$.)

3. We will revisit the original Fokker-Planck model for multilane traffic with lane-changes and explain why a multivalued fundamental diagram emerges in this case. A linear stability analysis for the corresponding equilibria will also be presented.

4. We will conclude with a general discussion on open problems and possible generalizations.

Bessel potentials and optimal Hardy and Hardy-Rellich inequalities

Nassif Ghoussoub
University of British Columbia

2:00 p.m. to 2:50 p.m.

We give necessary and sufficient conditions on a pair of positive radial functions V and W on a ball Ω of radius R in R^n, $n \geq 2$, so that the following inequalities hold for all $u \in C_0^\infty(\Omega)$:

$$\int_\Omega V(x)|\nabla u|^2 dx \geq \int_\Omega W(x)u^2 dx \tag{1}$$

and

$$\int_B V(x)|\Delta u|^2 dx \geq \int_B W(x)|\nabla u|^2 dx + (n-1)\int_B \left(\frac{V(x)}{|x|^2} - \frac{v'(|x|)}{|x|}\right)|\nabla u|^2 dx. \tag{2}$$

We then identify a large number of such couples (V, W) – that we call Bessel pairs – and the best constants in the corresponding inequalities. This will allow us to complete, improve, extend, and unify most related results –old and new– about Hardy and Hardy-Rellich type inequalities which were obtained by Caffarelli-Kohn-Nirenberg, Brezis-Vázquez, Adimurthi-Chaudhuri-Ramaswamy, Filippas-Tertikas, Adimurthi-Grossi-Santra, as well as some very recent work by Tertikas-Zographopoulos, Liskevich-Lyachova-Moroz, and Blanchet-Bonforte-Dolbeault-Grillo-Vasquez, among others.

This is joint work with Amir Moradifam.

The Infinity Laplacian: from classical analysis to image processing and random turn games

Adam Oberman
Simon Fraser University

2:50 p.m. to 3:40 p.m.

The Infinity Laplacian equation is currently at the interface of a number of different mathematical fields. It was first studied in the 1950s by the Swedish mathematician Gunnar Aronsson, motivated by classical analysis problem of building Lipschitz extensions of a given function. While Aronsson was able to find interesting exact solutions, progress stalled because solutions were non-classical. It took another forty years until analytical tools were developed to study the equation rigorously, and computational tools were developed which made numerical solution of the equation possible.

In the last decade, PDE theorists established existence and uniqueness, and (quite recently) appropriate regularity results. At the same time, the image processing community was using the operator for edge detection, and for inpainting, the reparation of images with damage. While the operator was promising, they had little success, since traditional methods for solving the equation yielded poor results.

It turns out that the right way to solve the equation is to go back to the original Lipschitz extension problem. This leads to a formula for the discrete operator with a simple interpreation, and good solution properties. This formula also leads to another surprising connection with probability theory.

Working in the unrelated field of percolation theory, a group of probalists (Peres-Shramm-Sheffield-Wilson) studying a randomized version of a marble game called Hex found a connection with the Infinity Laplacian equation.

This connection gives an interpretation of the equation as a two player random game.

I'll tell this story, and explain some of the more accessible properties of the equation, along with pictures and numerical results.

Landau-Lifshitz dynamics and Schroedinger maps

Stephen Gustafson
University of British Columbia

4:10 p.m. to 5:00 p.m.

The Landau-Lifshitz equations of ferromagnetism include, as a special case, a family of geometric evolution equations for maps into the 2-sphere, with the harmonic map heat-flow at one extreme, and the Schroedinger map equation (a geometric generalization of the linear Schroedinger equation) at the other. Understanding the global properties of these map evolution problems is a major challenge. I will describe some recent work concerning the questions of (possible) singularity formation, and of long-time asymptotics. This is joint work with Meijiao Guan, Kyungkeun Kang, and Tai-Peng Tsai.
