

Asymptotic behaviour of solutions to the fast diffusion equations

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Abstract

We will consider the equation $\frac{d}{dt}u = \Delta \log u$ on $\mathbb{R}^2 \times [0, T)$ with compactly supported initial data and discuss the rigorous proof of formal asymptotics established by J.R.King. We will show there are 2 different regions of behaviour of u , in the first one, after the appropriate rescaling we see a cigar $\frac{1}{(T/2)(|x|^2+1)}$ and in the second region we see the logarithmic cusp. We will also discuss the asymptotic behaviour of solutions to $\frac{d}{dt}u = \Delta u^{\frac{N-2}{N+2}}$ on \mathbb{R}^N (which is the Yamabe flow of the conformally flat metric on \mathbb{R}^N) and show that after the appropriate rescaling the solution converges to a Barenblatt.