Mathematics of Oil Exploration Workshop

PIMS-CINVESTAV

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FRACTAL SCATTERING OF WAVES FROM SOIL

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Using a combination of laboratory experiments and computer simulation, we show that microwaves reflected from and transmitted through soil have fractal dimension correlated to that of the soil's hierarchic permittivity network. The mathematical model relating the ground-penetrating radar record to the mass fractal dimension of soil structure is also developed. The fractal signature of the scattered microwaves correlates well with some physical and mechanical properties of soils. Oil Field Multiscalar fractal modellation of an oil field (Canterell, Gulf of Mexico) is presented. Minimum fractal dimension is at the cretaceous period, fractal dimension is correlated with resistivity, radioactivity, density and porosity.

Joint work with K. Oleschko, G. Korvin, R. Dario et al.
Volume 89, number 18 Physical Review Letters

PRICING AND HEDGING OF OIL EXPLORATION INVESTMENT PROJECTS

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Oil exploration investment projects valuation is the typical situation of pricing contingent claims that depend not only on tradable assets but also on other assets, which are not traded in the financial markets, which induce uncertainty. This is a particular case of incomplete markets for which the use of an extra criterion to select one price among the set of arbitrage-free prices is required.

In this talk, we explore the problem of pricing and hedging of derivative securities in incomplete markets due to insufficient number of assets available for investment (real options), we analyse the classical situation of using utility functions as selection criterion, and also the case of risk measures.

WAVE-EQUATION REFLECTION TOMOGRAPHY

Maarten de Hoop
Purdue University

Seismic data are commonly modeled by a linearization - leading to the introduction of a scattering operator - around a smooth background medium. The perturbation of the medium coefficient is assumed to contain discontinuities and results in reflections in the data. This leads to two inverse problems; first, the linearized inverse problem for the perturbation, and second, the estimation of the background, which is a priori unknown. We can establish whether the background velocity model is correct by testing whether the data are in the range of the scattering operator. This range can be characterized by pseudodifferential annihilators, which generalize the notion of differential semblance. (These annihilators can also be exploited for filling in `missing' data by source-receiver continuation.) A tool to construct annihilators and develop a method for wave-equation reflection tomography (addressing the second inverse problem) is the so-called wave-equation angle transform which is connected to the notion of data downward continuation. The angle transform maps surface reflection data into common image-point gathers revealing the redundancies in the data. The angle transform is, microlocally, invertible. By using particular curvilinear coordinates, generating a Riemannian metric, this can be established admitting the formation of caustics and `turning' rays. We will briefly discuss optimization strategies. In connection with these, we further develop a method of `velocity' continuation, transforming the above mentioned image gathers from an `initial' background `velocity' model to a `correct' model, and demonstrates that this can be formulated in terms of evolution equations. We will illustrate solving these evolution equations with the aid of a curvelet transform, as well as the `finite-frequencies' features of wave-equation reflection tomography.

In collaboration with C.C. Stolk, G. Uhlmann, and R.D. van der Hilst

THE ROLE OF OPTIMIZATION IN THE CHARACTERIZATION OF OIL RESERVOIRS TO FORECAST PRODUCTION

Susana Gómez
IIMAS, National University of Mexico

To forecast production, it is necessary to find the properties of the porous media of the reservoir under study, solving an inverse parameter estimation problem. These inverse problems are ill posed and highly non-linear, and usually have multiple solutions with good match to the pressure related data.

In this work we will discuss some characteristics of these inverse problems that may affect the performance and convergence of optimization methods.

Results for well test data characterization, using Evolutionary Algorithms and Newton type methods, will be used to illustrate these difficulties, and the characterization results of whole reservoirs (History Matching) using the Parallel Tunneling method will also be presented.

COMPRESSIVE SAMPLING MEETS SEISMIC IMAGING

Felix Herrmann
University of British Columbia

Seismic imaging involves the solution of an inverse-scattering problem during which the energy of (extremely) large data volumes is collapsed onto the Earth's reflectors. We show how the ideas from 'compressive sampling' can alleviate this task by exploiting the curvelet transform's 'wavefront-set detection' capability and 'invariance' property under wave propagation. First, a wavelet-vaguelette technique is reviewed, where seismic amplitudes are recovered from complete data by diagonalizing the Gramm matrix of the linearized scattering problem via the curvelet domain. Next, we show how the recovery of seismic wavefields from incomplete data can be cast as a compressive sampling problem, followed by a proposal to compress wavefield extrapolation operators via compressive sampling. Sampling in the Dirac basis and as well as in the modal domain are reviewed. During the latter, the measurement basis diagonalizes the extrapolation operator. In both cases, the wavefield extrapolation operators are reduced in size while the wavefields are recovered by employing curvelet-domain sparsity.

APPLICATIONS OF THE GABOR TRANSFORM IN SEISMIC IMAGING

Gary F. Margrave
University of Calgary

The Gabor transform, or windowed Fourier transform, is an effective technique to extend Fourier spectral theory to inherently nonstationary problems. A particularly simple formulation, based on a localizing window set constrained to form a partition of unity (POU), has proven very adaptable to seismic imaging applications. I will outline the Gabor theory and illustrate its connection to pseudodifferential operator theory. Then I will describe in detail the application to two problems in seismic image construction: deconvolution and migration. In the first case, we develop a Gabor multiplier that effectively corrects seismic data for both attenuation effects and source signature. The magnitude of the Gabor symbol of this operator is estimated from the data itself while the phase is constructed under the minimum phase assumption. In the second case, the problem of wavefield extrapolation in depth through laterally variable velocity is addressed through the construction of a non-uniform POU. This partition is constrained by an error criterion bounding lateral position error. The result is an effective pre-stack depth migration that generalizes directly to 3D. Both of these applications will be illustrated by data examples.

Collaborators: Michael P. Lamoureux (Professor of Mathematics), Carlos Montana (PhD candidate in geophysics), Yongwang Ma (PhD candidate in geophysics)

SOLUTION TO THE FUNDAMENTAL PROBLEM OF OIL EXPLORATION, AND SOLUTION FOR THE OPTIMIZATION AND DEVELOPMENT OF FIELDS NATURALLY FRACTURED

Fernando Olivera
PEMEX Exploración y Producción

The method of the boxes of Mandelbroot appears to determine that plane geometry (plane X, Y) of the fields in the basins has a behavior fractal. Nevertheless, since the maps of the fields is approximately one function of the radius of drains of wells and of the configuration of the structures, it is not possible to propose a well to drill that can be productive of oil or gas in an area obtained by the growth of a plane geometry fractal of the field. However, it was confirmed that the exact and excellent data are the geographical coordinates of producing wells of oil or gas as well as the depth of the producing interval. On any scale this allows the use Euclidean geometry to determine the laws of distribution of producing oil or gas wells. 25 theorems will appear that will lead upon presentment of applications of the Geofolds technology, which serves to find new producing reservoirs of oil or gas.

APPLICATIONS OF GENETIC ALGORITHMS TO LOGISTICS PROBLEMS

Martin Romero
Instituto Mexicano del Petróleo

We consider two logistic problems that are frequently encountered at oil companies. First, we study a scheduling problem with time restrictions. Then, we consider the pick up and delivery problem (PDP); for example, the helicopter offshore transportation of people for oil companies. For the PDP problem we present an algorithm based on two optimization techniques, a genetic algorithm and heuristic optimization. For both problems, we consider typical scenarios of relatively large numbers of participants. Our solutions are tested on examples with known optimal solutions.

Joint work with Sheremetov and Soriano (2007). Published in Theoretical Advances and Applications of Fuzzy Logic and Soft Computing by Castillo, Melin et al. Springer-Verlag.

PARABOLIC SCALING AND IMAGING

Hart Smith
University of Washington

Abstract: I will survey the mathematics of parabolic scaling, which is behind the construction of curvelet frames and Gaussian beam construction. The talk will explain why such frames are well suited for computing wave propagation of images with sharply defined fronts, and for the optimal compression of images with singularities along curves or hypersurfaces.

INVERSION FOR SEISMIC VELOCITIES

William Symes
Rice University

Output least squares inversion of seismic reflection data can take into account virtually any physics of seismic wave propagation, and produce highly detailed maps of subsurface elastic parameters. However the waveform inversion objective has many spurious local minima, so that convergence of Newton-type optimization algorithms requires very good initial model estimates, perhaps better than are commonly available. The seismic industry has developed an alternative approach, termed velocity analysis, for estimation of seismic velocities. We will explain the relation between velocity analysis and data-fitting inversion, and show how to formulate velocity analysis itself as a least-squares-type optimization with good global behaviour.

Venue: **Auditorio, Instituto de Biotecnología, Facultad de Ciencias Biológicas**

TIME	THURSDAY	FRIDAY
11:00 am	Hart Smith *University of Washington* **PARABOLIC SCALING AND IMAGING**	Susana Gómez *IIMAS, National University of Mexico* **THE ROLE OF OPTIMIZATION IN THE CHARACTERIZATION OF OIL RESERVOIRS TO FORECAST PRODUCTION**
12:00 pm	Fernando Brambila *Facultad de Ciencias, National University of Mexico* **FRACTAL SCATTERING OF WAVES FROM SOIL**	Maarten de Hoop *Purdue University* **WAVE-EQUATION REFLECTION TOMOGRAPHY**
1 :00 pm	William Symes *Rice University* **INVERSION FOR SEISMIC VELOCITIES**	Fernando Olivera *PEMEX Exploración y Producción* **SOLUTION TO THE FUNDAMENTAL PROBLEM OF OIL EXPLORATION, AND SOLUTION FOR THE OPTIMIZATION AND DEVELOPMENT OF FIELDS NATURALLY FRACTURED**
	Lunch	*Lunch*
4:30 pm	Myriam Cisneros-Molina *Programa de Matemáticas Aplicadas y Computación,Instituto Mexicano del Petróleo* **PRICING AND HEDGING OF OIL EXPLORATION INVESTMENT PROJECTS**	Felix Herrmann *University of British Columbia* **COMPRESSIVE SAMPLING MEETS SEISMIC IMAGING**
5:30 pm	Gary F. Margrave *University of Calgary* **APPLICATIONS OF THE GABOR TRANSFORM IN SEISMIC IMAGING**	Martin Romero *Instituto Mexicano del Petróleo* **APPLICATIONS OF GENETIC ALGORITHMS TO LOGISTICS PROBLEMS**