ABSTRACTS
SYMPOSIUM ON KINETIC EQUATIONS AND METHODS
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Eric Carlen, Georgia Institute of Technology

Title: Determination of the spectral gap in the Kac model for physical momentum and energy conserving collisions.

Abstract: The Kac model considered here models the collisions of N particles with three-dimensional velocities by a random walk in which the steps correspond to binary collisions that conserve momentum as well as energy. The state space of this walk is a sphere of dimension $3N-4$. The Kac conjecture concerns the spectral gap in the one step transition operator Q for this walk. In this paper, we compute the exact spectral gap for two types of the collision kernel.

As in previous work by Carlen, Carvalho and Loss where a lower bound on the spectral gap was proved, we use a method that relates the spectral properties of Q to the spectral properties of a simpler operator P , which is simply an average of certain non-commuting projections. The new feature is that we show how to use a knowledge of certain eigenfunctions and eigenvalues of P to determine spectral properties of Q, instead of simply using the spectral gap for P to control the spectral gap for Q, inductively in N , as in previous work. The methods developed here can be applied to many other high-dimensional stochastic process, as we shall explain. We also use some deep results on Jacobi polynomials to obtain the required spectral information on P , and we show how the identity through which Jacobi polynomials enter our problem may be used to obtain new bounds on Jacobi polynomials. This is joint work with Michael Loss and Jeffery Geronimo.

Jose Carrillo, Universitat Autònoma de Barcelona

Title: Long time asymptotics for the Aizemann-Bak model

Abstract: The Aizenman-Bak model for reacting polymers is considered for spatially inhomogeneous situations in which they diffuse in space with a non-degenerate size-dependent coefficient. Both the break-up and the coalescence of polymers are taken into account with fragmentation and coagulation constant kernels. We demonstrate that the entropy-entropy dissipation method applies directly in this inhomogeneous setting giving not only the necessary basic a priori estimates to start the smoothness and size decay analysis in one dimension, but also the exponential convergence towards global equilibria for constant diffusion coefficient in any spatial dimension or for non-degenerate diffusion in dimension one. We finally conclude by showing that solutions in the one dimensional case are immediately smooth in time and space while in size distribution solutions are decaying faster than any polynomial. Up to our knowledge, this is the first result of explicit equilibration rates for spatially inhomogeneous coagulation-fragmentation models.

Walter Craig, McMaster Univeristy

Title: Invariant tori for Hamiltonian PDE's.

Abstract: A Hamiltonian PDE can be written in the form

$$\partial_t v = J \mathrm{grad}_v H(v) \ , \qquad J^T = -J \ , \quad H(v) : B \to \mathbb{R} \ ,$$

where the function space B is chosen appropriately for the particular problem at hand. Denote by $\Phi_t(v)$ the flow of this system. An invariant manifold $M \subseteq B$ satisfies $\Phi_t(M) \subseteq M$ for all times t. Invariant manifolds which are Lagrangian or isotropic tori play an important rôle in Hamiltonian systems. An invariant embedded m-dimensional invariant torus $\mathbb{T}^m$ in the phase space B with the structure of a linear flow with frequency vector Ω is given by a mapping

$$S : \mathbb{T}^m \to B \ , \qquad \Phi_t(S(\xi)) = S(\xi + t\Omega) \ .$$

This implies that the embedding $S(\xi)$ must satisfy the PDE

$$J^{-1}\Omega \cdot \partial_\xi S - \mathrm{grad}_v H(S) = 0 \ .$$

This problem can be posed as a variational principle with Lagrange multiplier Ω, whose Euler – Lagrange equations involve a small divisor problem. This talk discusses progress over the past number of years on solutions of this equation, for various examples of Hamiltonian PDEs, in cases of nonresonant and resonant tori.

Tai-Ping Liu, Stanford

Title: Solving Boltzmann Equation, Green's Function Approach.

Abstract: We study the pointwise properties of the Green's function for the linearized Boltzmann equation. This is useful for the study of nonlinear waves and the boundary effects. We will illustrate this approach with examples on shock waves and the initial-boundary value problem for the full Boltzmann equation.

Peter Markowich, University of Vienna, RICAM Linz

Title: Nonlinear diffusions as limits of BGK-type kinetic equations

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Benoit Perthame, ENS Paris

Title: Strichartz estimates for kinetic equations.

Abstract: This talk will present several types of results expressing 'regularizing effects' for kinetic transport equations and compare them (dispersion, averaging, moments, Strichartz).

We will use these tools in the framework of the Alt-Dunbar-Othmer kinetic model for chemotaxis (cell motion induced by an external chemical emitted by the cells themselves) we will show how they can be used and what kind of nonlinearity can be reached.

Fraydoun Rezakhanlou, UC Berkeley

Title: Coagulating Brownian particles and moment bounds for Smoluchowski's Equation.

Abstract: Smoluchowski's equation is used to describe the coagulation process macroscopically. Microscopically clusters of various sizes coalesce to form larger clusters. I describe how from a stochastic model of interacting Brownian particles one can derive Smoluchowski's equation in a scaling limit. This is achieved only if there is no gelation. I also discuss some new bounds on the solutions to Smoluchowki's equations. This allows us to show uniqueness and the conservation of mass under suitable assumptions on the diffusion rates and the coagulation propensities.

Walter Strauss, Brown University

Title: Stability Criteria for the Vlasov-Maxwell System.

Abstract: For the relativistic Vlasov-Maxwell system that models a plasma, many equilibria are stable and many are unstable. In this talk I will consider axisymmetric equilibria of the form $f(e,p)$ that are decreasing in the particle energy e and also depend on the particle angular momentum p. Then a necessary and sufficient condition for linear stability is the positivity of a certain linear operator $\mathcal{L}^0$. This operator $\mathcal{L}^0$ is relatively simple, much simpler than the generator of the full linearized system. It has an interesting non-local term that can definitely affect its positivity.

There is a similar reduction in the simpler case of 1.5 dimensional symmetry. In the important example of a purely magnetic equilibrium, explicit conditions for the linear/nonlinear stability/instability are obtained.

This talk represents joint work with Zhiwu Lin.

Catherine Sulem, University of Toronto

Title: Water waves over a varying bottom.

Abstract: I will discuss the problem of nonlinear wave motion of the free surface of a body of fluid with a varying bottom. The object is to describe the character of wave propagation in a long-wave asymptotic regime. The case in which the bottom topography is periodic on small scales is shown to homogenize completely, and we can compute how the bottom irregularities affects the effective Boussinesq equations and in the appropriate unidirectional limit, the Korteweg de Vries equation.

In the case of a bottom described by a random, stationary ergodic process with sufficiently strong mixing conditions, we show that the problem does not homogenize fully. In the long wave limit, the random effects are governed by a canonical limit process which is equivalent to a white noise through Donsker's invariance principle, with only one free parameter, the variance. The KdV limit is derived, while at the same time the random effects and the degree of scattering due to the variable bottom are quantified.

This is a joint work with A. de Bouard, W. Craig and P. Guyenne.

Shigeru Takata, Kyoto University

Title: Knudsen compressor and gas separation.

Abstract: In this talk, we shall report a new feasibility of the Knudsen compressor as a gas separator. After a brief introduction of the Knudsen compressor and its physical mechanism, we proceed to the derivation of a fluid-dynamic model (from the Boltzmann equation) that describes the behaviour of the mixture in the compressor. Then, we present the results of numerical simulation of this model which manifest that the Knudsen compressor works certainly as a gas separator. Our simulation was carried out for various molecular models, not only for fundamental models, the hard-sphere and Maxwell molecules, but also for more realistic models such as the inverse power-law potential and Lennard-Jones models, assuming the McCormack model equation at the kinetic level. We shall also show that the modelling by the celebrated Maxwell molecule (or the BGK-type model equation) fails to capture the phenomenon of the gas separation in the device. This presents a remarkable contrast to the capability of the other fundamental model, the hard-sphere molecule, even though this model exaggerates the phenomenon to some extent. This talk is based on the paper with Hiroshi Sugimoto and Shingo Kosuge that was published in Eur. J. Mech., B/Fluids (2007).
