# The Parabolic-Hyperbolic System of Fluid-Structure Interaction: Semigroup Well-Posesdness, Spectral Analysis, Strong and Exponential Stability, and Backward Uniqueness

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**Abstract**

In 3-d or 2-d, we consider an elastic structure (described by the system of dynamic elasticity) immersed in a fluid (Navier-Stokes), with coupling taking place at the interface. In this preliminary analysis, the elastic structure is fixed but vibrates. In the case of the linear Navier-Stokes, we shall present the following problems (joint work with George Avalos, University of Nebraska):

- semigroup well-posedness in the natural energy space, with explicit generator;
- spectral analysis of the system (and its adjoint);
- analysis of strong stability;
- exponential stability with dissipation at the interface;
- backward uniqueness of the resulting semigroup.

The resolvent of the generator is not compact (it is compact only on the velocity and fluid components, but not in the position component). Spectral analysis depends on a geometric property of the structure (that is, on whether or not the structure is partially flat). More

precisely: the origin is always an eigenvalue with a one-dimensional explicit eigenspace. If the structure is partially flat, this is the only eigenvalue, indeed the only point of the spectrum, in the closed right-hand of the complex plane. In general, e.g., in the case where the structure is a sphere, the spectrum of the generator has also a countable (explicit) sequence of eigenvalues on the imaginary axis. Thus, at best, the problem is strongly stable, but only the energy state space factored out by the one-dimensional eigenspace of the origin.

When a dissipation term is introduced at the interface, then exponential stability can be established, this time with no geometrical conditions imposed on the structure. Finally, we show backward uniqueness of this parabolic-hyperbolic coupled system: If the semigroup solution vanishes at some finite time, then the initial condition must be zero.