A further property of the Cosserat operator and its application to regularity of weak solutions of Stokes' problem and related equations

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Abstract

For sake of simplicity we regard bounded domains $G \subset^n$ ($n \geq 2$) with sufficiently smooth boundaries ∂G. Let for $1 < q < \infty$

$$L_0^q(G) := \left\{ p \in L^q(G) : \int\limits_G p dy = 0 \right\}.$$

For $p \in L_0^q(G)$ we regard weak solutions $\underline{v} = (v_1, \ldots, v_n) \in \underline{H}_0^{1,q}(G) := \underline{H}_0^{1,q}(G)^n$ of the weak n Dirichlet problems

$$\sum_{i,k=1}^{n} \int\limits_G \partial_i v_h \partial_i \phi_k =: \langle \nabla \underline{v}, \nabla \underline{\phi} \rangle = \langle p, \underline{\phi} \rangle := \int\limits_G p \underline{\phi} \tag{1}$$

$\left(\underline{\phi} \in \underline{H}_0^{1,q'}(G), q' := \frac{q}{q-1} \right)$.

The operator $Z_q : L_0^q(G) \to L_0^q(G)$, $Z_q p := \underline{v}$ where $\underline{v}$ is the unique solution of (1), is called the Cosserat operator. In the sense of a direct ($q = 2$ orthogonal) decomposition it holds

$$L_0^q(G) = A^q(G) \oplus B_0^q(G)$$

where

$$A^q(G) := \left\{ \Delta s : s \in H_0^{2,q}(G) \right\}$$

and

$$B^q(G) := \left\{ p \in L_0^q(G) : \langle p, \Delta s \rangle = 0 \quad \forall s \in H_0^{2,q'}(G) \right\}.$$

This decomposition is equivalent to the weak Dirichlet problem in L^q for Δ^2. The study of the properties of $Z_q \mid_{A^q(G)}$ is simple. By means of regularity properties for the Laplacian and the use of an ansatz by M. Crouzeix recently St. Weyers succeeded in proving nice properties of the operator $(Z_q - \frac{1}{2}I) \mid_{B_0^q(G)}$. On the basis of one of his results it is possible to deduce all regularity properties of solutions of Stokes', Stokes-like and Lamé's equation solely from the well-known regularity results for the (scalar) Laplace and Bilaplace equation. Analogous results we achieved for exterior domains too.