Spatial behavior of solutions of the Navier-Stokes equations in aperture domains

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Abstract

In this paper, we consider the initial boundary value problem for the Navier-Stokes equations in an *aperture domain* Ω, cf. Heywood 1976, Solonnikov 1977, Galdi 1994, Farwig 1996, Hishida 2004. We are interested to show spatial asymptotic behavior of solutions corresponding to a flux $\phi(t)$ (here $\phi(t)$ denotes the flux through the aperture M, that is 2-dimensional manifold). The flux $\phi(t)$ is assumed only bounded for $t > 0$. In paper by Crispo Maremonti it is proved that to *small* fluxes $\phi(t)$ and to *small* initial data there correspond, defined for any $t > 0$, smooth solutions of the Navier-Stokes problem in an aperture domain Ω. Here we study the spatial behavior of these solutions in two special cases of zero initial data and when the data have a suitable spatial decay rate.

Introducing a cut-off function, we give a suitable decomposition of the initial boundary value problem in Ω. Hence we reduce our problem to the case of an half-space. By making use of the Green formula of Stokes problem in half-space, given by Solonnikov 2003, we are able to prove the spatial behavior uniformly in t for solutions to Stokes problem. From one side, this approach seems a simple way to attach the problem. From another side, it leads to a very careful estimates related to exact solutions expressed through the Green formula in half-space, they trace back to the estimates of the Green function already proved by Solonnikov 2003.

Our matter is of correctly posing the well posedness problem for non stationary Stokes and Navier-Stokes equations in an aperture domain when the flux Φ is a function of time `only bounded`. To be specific, we consider the initial boundary value problem, and restric our attention to solutions which possess at least enough regularity for the development of uniqueness and stability theory. Since the work of Finn and Heywood, it is well known that, in unbounded domains, boundary conditions for the velocity and the flux do not suffice to determine unique solutions. Actually further decay conditions at infinity are necessary. Problem of decay of solutions at infinity has been studied by means of methods of functional analysis, however these tools require summability in time on the flux through the aperture. Our aim in this paper is to construct a class of solutions with the velocity $\mathbf{u}$ decaying to zero at infinity as $|x|^{-2}$, while at initial time $\mathbf{u}_0$ is required to decay as $|x|^{-2}$. In particular, for our results the initial velocity need not to be of square summable, a point of interest in unbounded domains.

Our existence theorems appear new also for the linear Stokes equations, and proofs are based on sharp estimates on the explicit representations of velocity and pressure through Green functions [Solonnikov]. One of primary purpose in using potential theoretic methods has been to make possible the consideration of non L^2 initial values and flux, in space and time respectively. We take advantage of the geometry of the domain, and obtain our sharp decay estimates, and we may solve the problem of attaining steady solutions as limit of non stationary ones, and of proving existence, and attainability of periodic flows. In conclusion our existence results of non stationary solutions to the Stokes and Navier-Stokes equations appear to be new in several respects: decay rate in aperture domains is sharp; the hypotheses on the data allow non summability on initial data and on the flux, existence, and attainability of periodic flows is proved.

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