Variational Multiscale Methods in Computational Fluid Dynamics: Recent Progress and Challenges with Emphasis on the Development of Turbulence Models

Tom Hughes
University of Texas at Austin
hughes@ices.utexas.edu

Abstract

I will discuss the development of the variational multiscale (VMS) approach for solving partial differential equation systems arising in fluid dynamics. The background is this: Any reasonable method utilizing functions capable of resolving the exact solution will obtain it. However, in numerical analysis we typically employ finite-dimensional spaces of functions that are unable to give good approximations for many fluid dynamical problems of practical interest. In addition, the basic variational methods (e.g., Galerkin)are not as reasonable as often assumed in the finite-dimensional setting. Stability, present in the continuous setting, is often not inherited for typically utilized finite-dimensional subspaces. The possibilities are to improve the function spaces, improve the variational methods, or both. Improving the spaces is possible but difficult. It has been a pathway to attaining stability for simpler problems. For more complex problems, enhancing the stability of the variational method, without upsetting its consistency, has been a more practical direction. This is the essence of so-called stabilized methods. But stability is not the only issue in computational fluid dynamics (CFD). In modeling turbulence, the ef-

fects of unresolved scales on resolved scales must also be accounted for.

VMS is a paradigm that derives directly from the variational formulation of the partial differential equations. In very simple situations it coincides with stabilized methods, but in more complicated cases it is richer. In addition to providing additional stability, it accounts for the effects of unresolved scales, and thus is a general framework for turbulence modeling as well as the derivation of CFD methods.

Recent research in VMS concerns comparisons with stabilized methods, calculation of the fine-scale Green's function, the use of continuous and discontinuous function spaces, minimizing the error in various measures, approximating discontinuities, the fine-scale field as error estimator, geometrically inspired approximations, weak boundary conditions, and turbulence modeling. In my talk I will sample from some of these recent developments and emphasize the use of VMS as a theoretical means for developing turbulence models. In particular, I will present a formulation of LES that is derived entirely from the Navier-Stokes equations without recourse to any external ad hoc devices, such as eddy viscosity models, and I will demonstrate the effectiveness of the ideas through numerical examples.