The Navier-Stokes flow around a rotating obstacle with time-dependent body force

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Abstract

We study the motion of a viscous incompressible fluid filling the whole 3-dimensional space exterior to a rigid body, that is rotating with constant angular velocity ω, under the action of external force. By using a frame attached to the body, the equations are reduced to

$$\partial_t u + u \cdot \nabla u = \Delta u + (\omega \times x) \cdot \nabla u - \omega \times u - \nabla p + f, \qquad \text{div } u = 0$$

in a fixed exterior domain D with boundary condition $u|_{\partial D} = \omega \times x$. Given $f \in BUC(\mathbb{R}; \dot{W}^{-1}_{3/2,\infty}(D))$, we consider this problem in $D \times \mathbb{R}$ and prove that there exists a unique solution $u \in BUC(\mathbb{R}; L_{3,\infty}(D))$ when f and $|\omega|$ are sufficiently small. If, in particular, the external force for the original problem is independent of t, then f is periodic with period $2\pi/|\omega|$. In this situation, as a corollary of our result, we obtain a periodic solution with the same period. We also discuss the stability of the obtained solution with respect to small initial disturbance in $L_{3,\infty}(D)$.