Optimal control problems for Navier-Stokes system

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Abstract

Two optimal control problems for Navier-Stokes equations are considered. The first is optimal boundary control problem for the three-dimensional, evolutionary Navier-Stokes equations in an exterior of bounded domain Ω. The control objective is to minimize the drag functional, and control is effected through the Dirichlet boundary condition on $\partial\Omega$.

First of all we find the proper space of vector-fields on boundary $\partial\Omega$ where we look for the control. Adding the norm of this space as a term of the cost functional we obtain well-posedness of considered optimal control problem. The existence of an optimal solution is proved. A strong form of an optimality system of equations is derived.

These results are based on a specially created theory of boundary value problems for Navier-Stokes equations with non zero boundary condition belonging to a non standard space of traces. Derivation of optimality system is based also on regularity results for the adjoint Oseen equations with regular initial data which do not satisfy the compatibility conditions.

The second problem is devoted to optimization of 2D steady-state Navier-Stokes equations in a special bounded domain Ω with a control defined on a part Γ of the boundary $\partial\Omega$. The feature of this problem is that we consider as a control u the boundary conditon "'adjoint"' to Dirichlet boundary condition: $u = (\partial_n v - pn)|_\Gamma$ where v is velocity

of fluid flow, p is its pressure (both are defined in Ω), and n is an external normal to the curve Γ.

This control is more natural from the point of view of applications than Dirichlet control, but in order to solve corresponding optimization problem one has to overcome a number of mathematical difficulties which will be discussed in a talk.

Existence of solution for this optimal control problem is proved and its optimality system is derived.

The first optimal control problem has been solved together with M.Gunzburger and S.Hou [1], and the second problem is investigated in collaboration with R.Rannacher.

[1] A.Fursikov,M. Gunzburger and L. Hou. *Optimal boundary control for the evolutionary Navier-Stokes system: the three-dimensional case.*- SIAM. J. Control Optim. v.43, #6, (2005),2191-2232.