Numerical methods for the simulation of fluid-structure interaction

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Abstract

In these four lectures we present numerical methods for simulating the interaction of viscous liquids with rigid or elastic bodies. General examples of fluid-solid/structure interaction (FSI) problems are flow transporting rigid or elastic particles (particulate flow), flow around elastic structures (airplanes, submarines) and flow in elastic structures (hemodynamics, transport of fluids in closed containers). In all these settings the dilemma in modeling the coupled dynamics is that the fluid model is normally based on an Eulerian perspective in contrast to the usual Lagrangian formulation of the solid model. This makes the setup of a common variational description difficult. However, such a variational formulation of FSI is needed as the basis of a consistent Galerkin discretization with a posteriori error control and mesh adaptation, as well as the solution of optimal control problems based on the Euler-Lagrange approach.

A common approach to dealing with this problem is to separate the two models, solve each separately, and so converge iteratively to a solution which satisfies both together with the interface conditions. Solving the separated problems serially multiple times is referred to as a “partitioned approach”. A partitioned approach does not contain a variational equation for the fluid-structure interface. To achieve this, usually an auxiliary unknown coordinate transformation function is introduced for the fluid domain. With its help the fluid problem is rewritten as one on the transformed domain which is fixed in time. Then, all computations are done on the fixed reference domain and as part of the computation the auxiliary transformation function has to be determined at each time step. This is the so-called “arbitrary Lagrangian-Eulerian” (ALE) method.

Both, the partitioned and the transformation approach to overcome the Euler-Lagrange discrepancy explicitly track the fluid-structure interface by mesh adjustment and are generally referred to as “interface tracking” methods. Both methods leave the structure problem in its natural Lagrangian setting. An alternative approach is posing the fluid as well as the structure problem in a fully Eulerian framework. In the Eulerian setting a phase variable is employed on the fixed mesh to distinguish between the different phases, liquid and solid. This approach to identifying the fluid-structure interface is generally referred to as “interface capturing”, a method commonly used in the simulation of multiphase flows. Examples for the use of such a phase variable are the Volume of Fluid (VoF) method and the Level Set (LS) method. In the classical LS approach the distance function has to continually be reinitialized, due to the smearing effect by the convection velocity in the fluid domain. This makes the use of the LS method delicate for modeling FSI problems particularly in the presence of cornered structures. To cope with this difficulty a variant of the LS method is proposed that makes reinitialization unnecessary. We describe these variational methods for simulating two prototypical types of fluid-solid/structure interaction phenomena: the movement of rigid bodies in a viscous fluid and the interaction of a viscous fluid with an elastic structure.

The first lecture discusses general aspects of the numerical simulation of fluid-structure interaction. The contents are "solution approaches", "variational formulations", "ALE methods", "Eulerian methods", "Galerkin finite element approximation", "a posteriori error estimation", and "mesh adaptation".

The second lecture concerns the special situation of fluid-'single rigid body' interaction, which requires very specialized numerical approaches. The contents are "body frame", "stationary free-fall problem", "numerical approximation", "domain truncation", "economical meshes", "hydrodynamic stability", and "nonstationary free fall".

The third lecture treats the case of fluid-'many rigid bodies'-wall interaction. The contents are "stress-DLM method", "fractional-step scheme", "finite element approximation", and "test examples".

The fourth lecture considers fluid-'elastic structure' interaction. The contents are "Eulerian variational formulation", "numerical approximation", "mesh adaptation", "large body deformation", and "test examples".