Hamiltonian systems of hydrodynamic type

Patrick Reynolds
Mathematics & Statistics, Queens University
reynolds.ap@gmail.com

Abstract

Systems of hydrodynamic type (that is, quasi-linear systems of PDEs) are written as $u^i_t(t,x) = \sum_{j=1}^n A^i_j(u(t,x))u^j_x(t,x)$, for $i = 1,\ldots,n$. Think of u^i as coordinates in R^n; under a change of coordinates, A^i_j transforms as a $(1,1)$-tensor, and this fact hints that classical differential geometry has an important role to play in the theory of such PDEs. I'll present the Hamiltonian formalism for systems of hydrodynamic type, with emphasis on a classification theorem (due to Dubrovin and Novikov) for non-degenerate Poisson brackets of hydrodynamic type, which hinges on flat pseudo-Riemannian metrics and their compatible connections on n-dimensional manifolds.