Applications of term structure models

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Section 1. Dynamic hedging

Section 2. Option valuation and investment

Section 1. Dynamic hedging

- Dynamic hedge :

  Relies on the use of short-term instruments (futures contracts) to cover long-term forward commitments on the physical market
- Reasons :

  - Actively traded futures contracts have a short maturity

  - Insufficient maturity of traded contracts
- Constraints :

  - Find the adequate hedge ratio

  - Rebalance the hedge portfolio when the futures' expiration date approaches

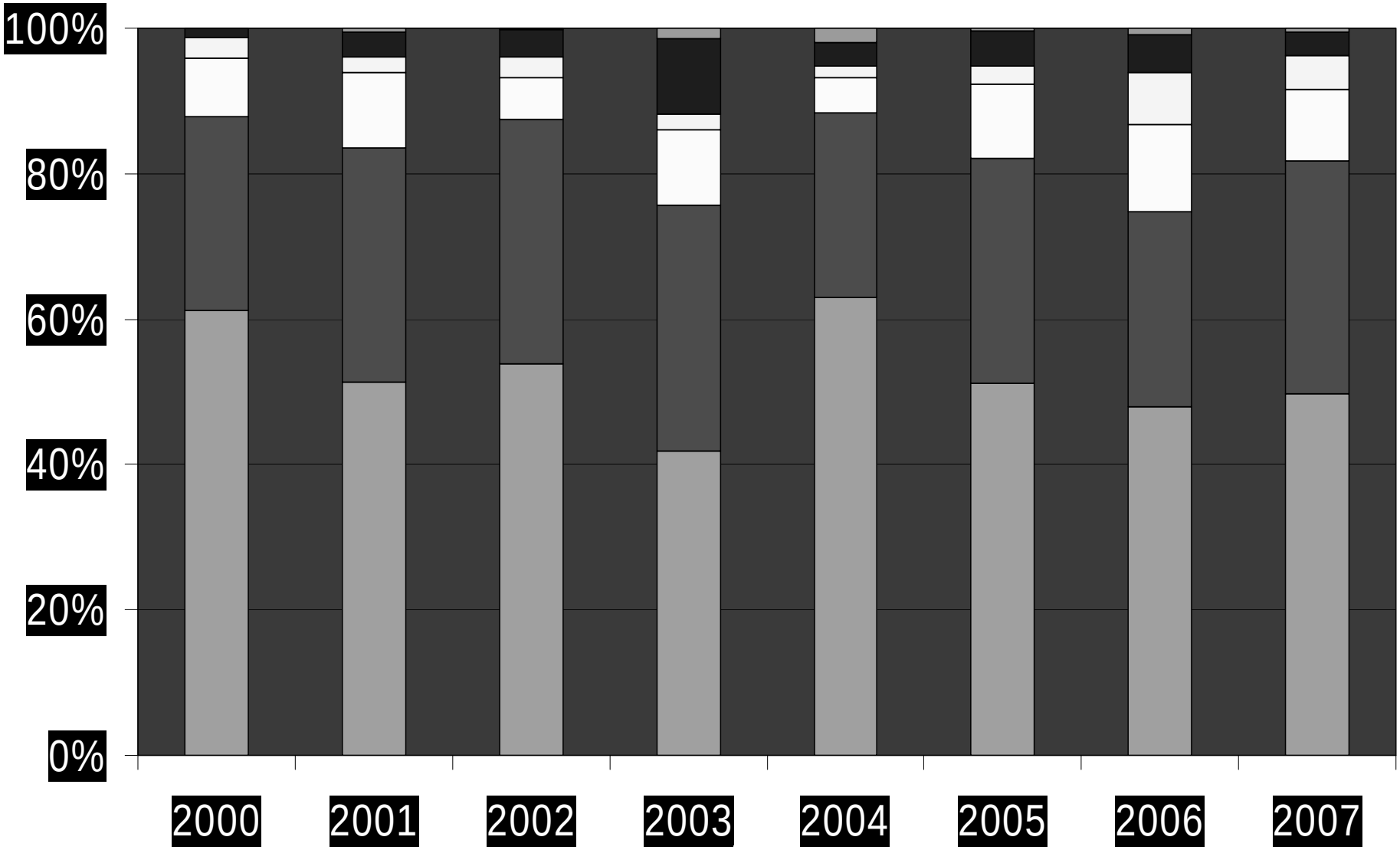
  Dynamic hedging entails rollover basis risk
- Question : Are dynamic hedge reliable?

Liquidity of commodity futures markets

- Maximum maturity :

  7 years, on the crude oil market (for futures only)
- Liquidity is usually concentrated on the first three maturities
- Open interest is also concentrated on the short-term
- Same phenomenon on :
  - Interest rates
  - Currencies (99% of transactions < 1 year)

Crude oil futures market (Brent)



An exemple of dynamic hedging : Metallgesellschaft

The facts:

- In 1994, on the American petroleum market
- MG proposes forward sales of petroleum products for a delivery in 5 or 10 years

$$F(t,T)$$

- MG does not hold petroleum products
- MG intends to wait until delivery to buy these products on the spot market

$$S(T)$$

- Risk of a rise in the prices of petroleum products

The hedge

- **Stack and roll strategy**
- Stack :

  Use of the nearest futures contracts to hedge the forward commitments

- Roll :

  The hedge must be rebalanced regularly

- Hedge ratio :

  100%

The results

- Commercially, at first,…

  …it was a tremendous success

- Financially, in the end,…

  … 2.4 billions dollars were lost

- The whole firm was restructured
- The MG case initiated researches on dynamic hedging in commodity markets

General features

- The dynamic hedging strategies rely on term structure models
- The differences between the strategies are due mainly to divergences in the assumptions concerning the behavior of commodity prices
- The hedge ratios:
  - depend on the state variables of the model
  - are an inverse function of the maturity of the commitment

- **The hedge portfolios**

  - are constituted of a number of positions which is superior or equivalent to the number of state variables

  However:

  - The convenience yield is not a traded asset
  - The long term price is not a traded asset
  - Transaction costs are high in the physical market

  Thus, the hedge portfolio is a combination of futures contracts having different maturities

  It does not rely on an investment in the underlying asset

Hedge ratios for Schwartz' model (1997)

- Two positions for delivery in t_1 and t_2:

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$$\begin{cases} w_1 F_S(S, C, \tau_1) + w_2 F_S(S, C, \tau_2) = e^{-r\tau} F_S(S, C, \tau) \\ w_1 F_C(S, C, \tau_1) + w_2 F_C(S, C, \tau_2) = e^{-r\tau} F_C(S, C, \tau) \end{cases}$$

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  - Solution of the system:

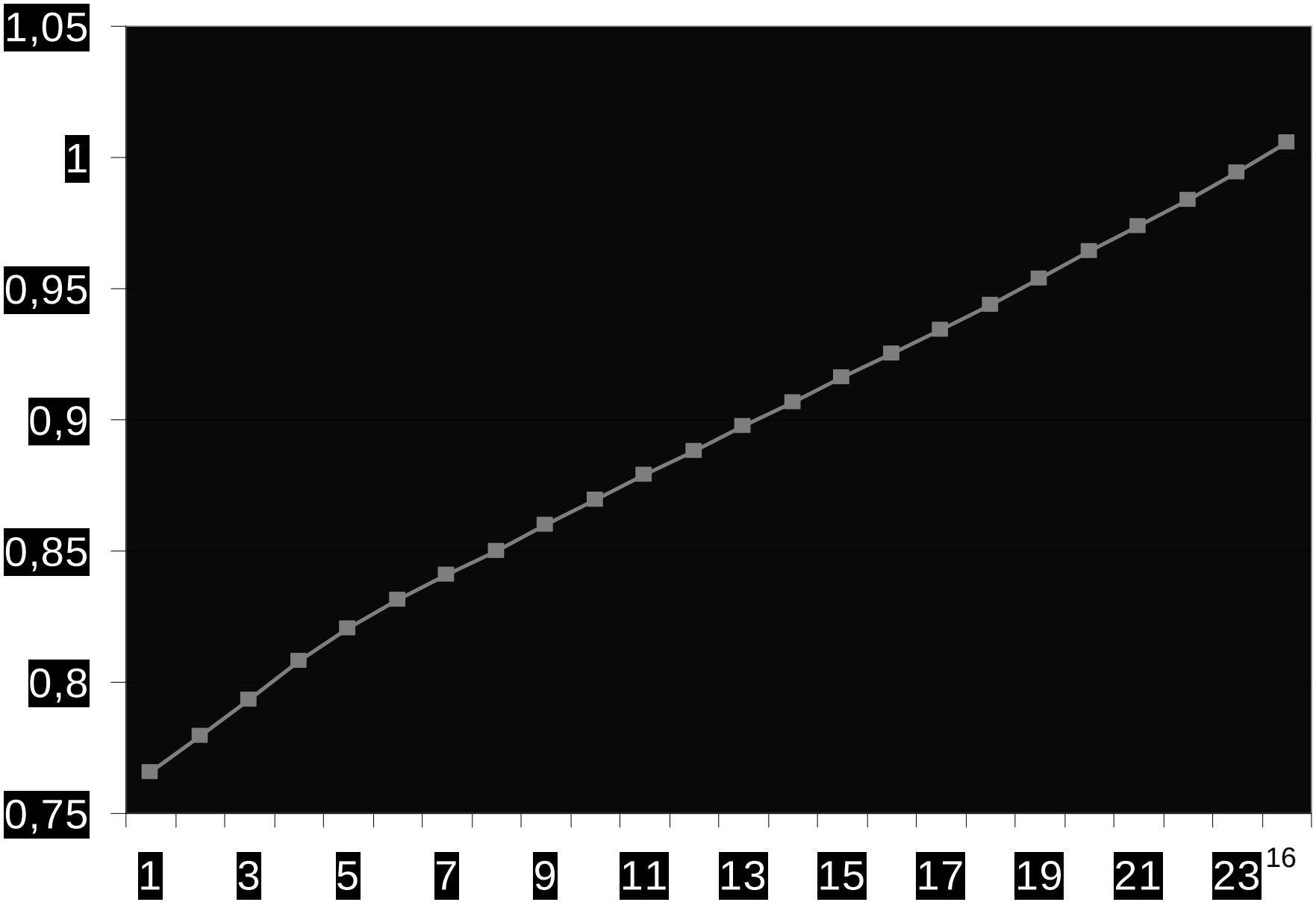
$$w = e^{-r\tau} Y(\tau) \begin{bmatrix} \dfrac{H(\kappa,\tau_2) - H(\kappa,\tau)}{(H(\kappa,\tau_2) - H(\kappa,\tau_1))Y(\tau_1)} \\ \dfrac{H(\kappa,\tau) - H(\kappa,\tau_1)}{(H(\kappa,\tau_2) - H(\kappa,\tau_1))Y(\tau_2)} \end{bmatrix}$$

Where:

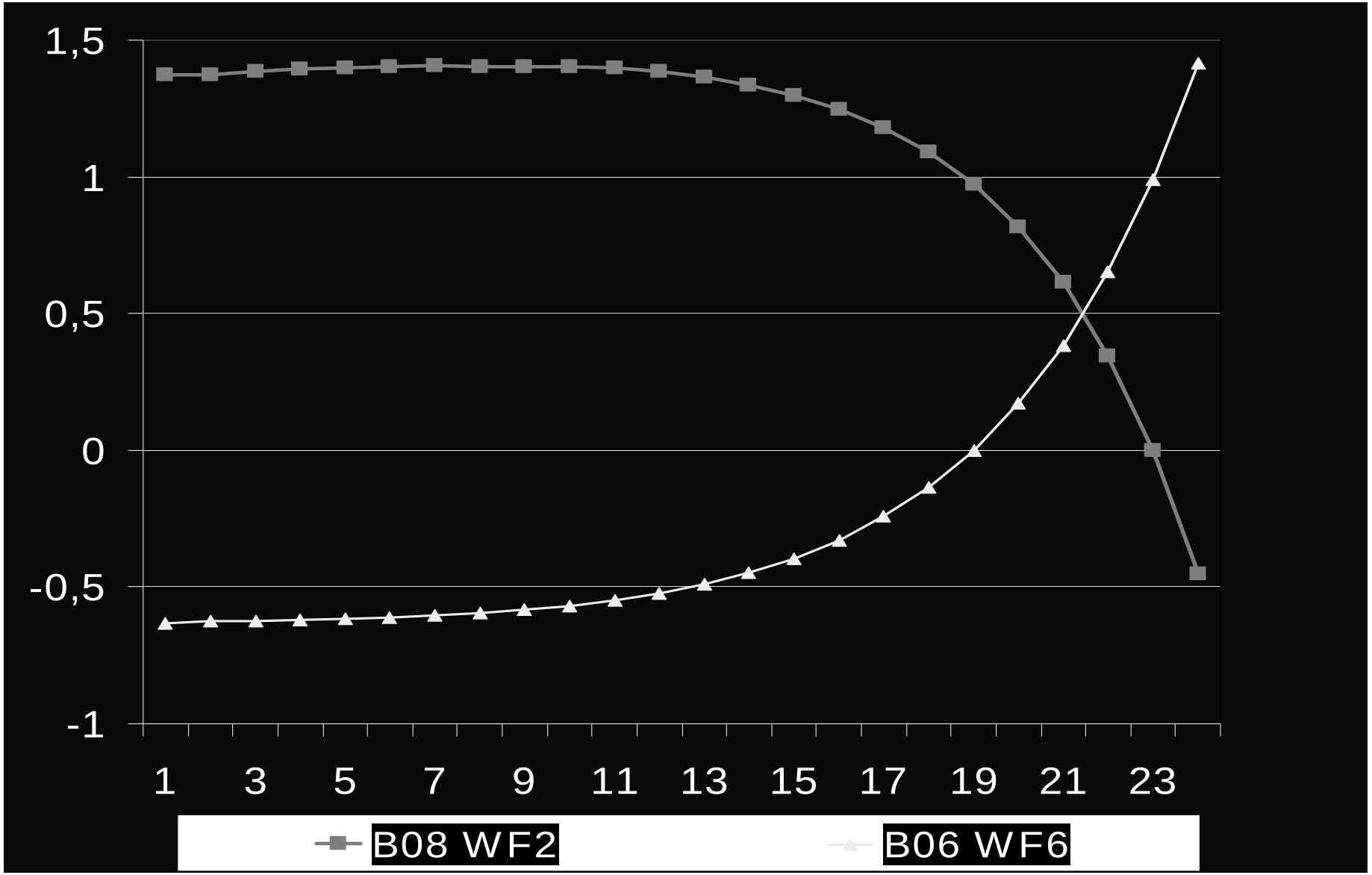
$$Y(\tau_i) = F_S(S, C, \tau_i)$$

$$H(\kappa, \tau_i) = \frac{1 - e^{-\kappa \tau_i}}{\kappa}$$

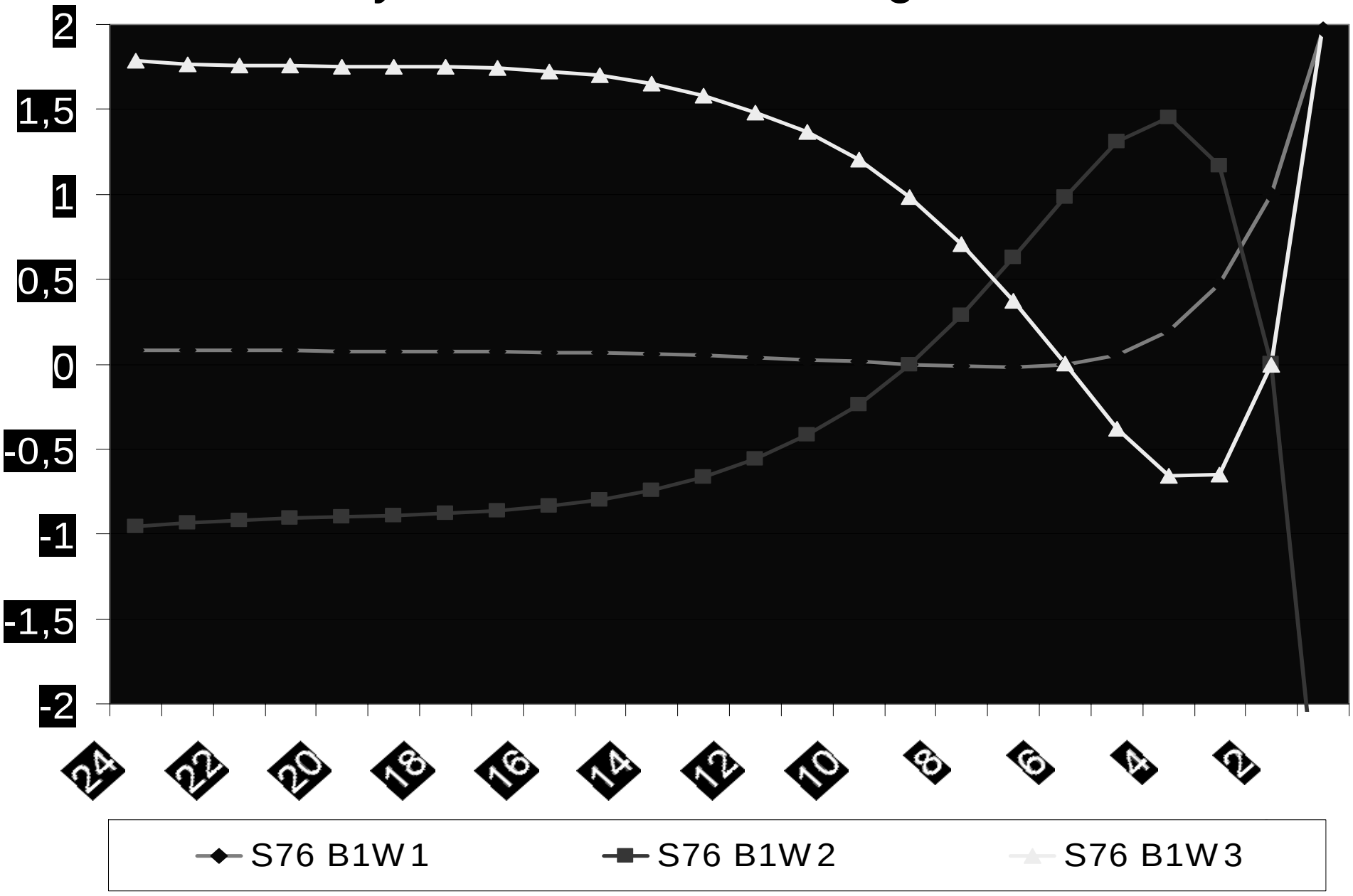
Hedge ratios, one-factor model
Forward = 2 years, Roll = 1M, Hedge = 2 months



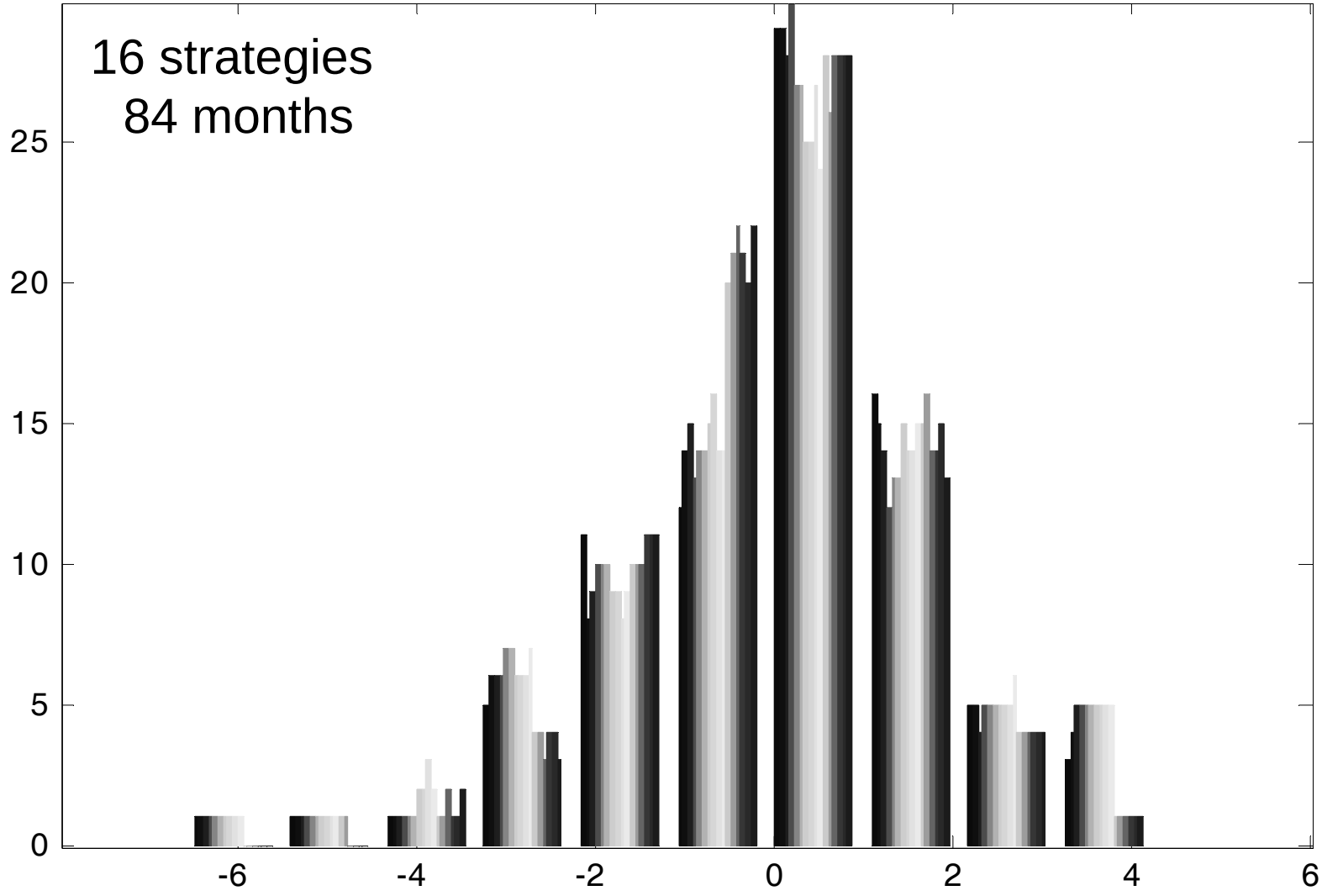
Hedge ratios, two-factor model
Forward = 2 years, Roll = 1 M, Hedge = 2 & 6M



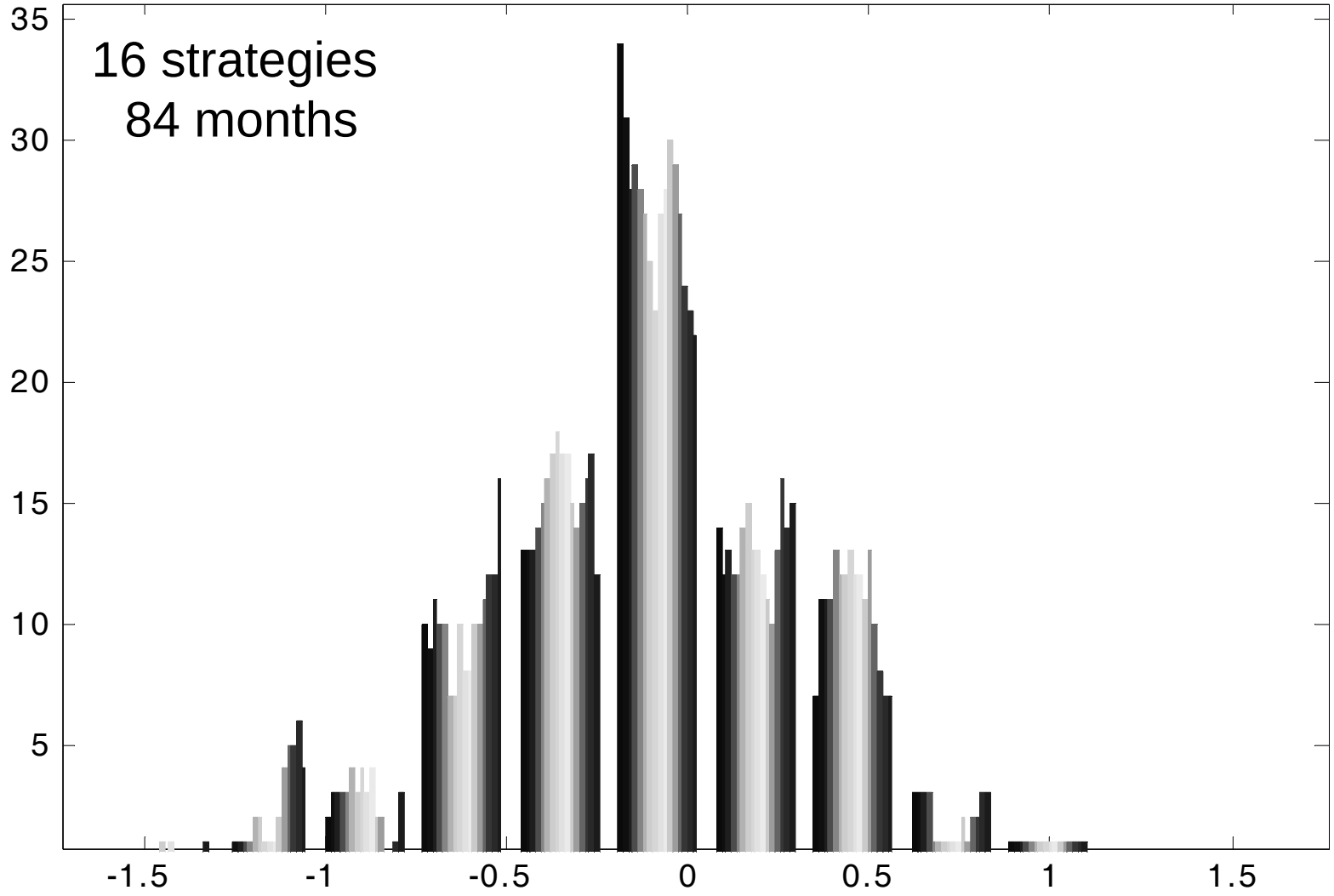
Hedge ratios, three-factor model, Forward = 2 years, Roll = 1M, Hedge = 2 & 6 & 9 M



Two factor model,
Forward = 7 years, Roll = 1M, Hedge = 2 & 6 M



Two factor model,
Forward = 7 years, Roll = 1M, Hedge = 2 & 24 M



- **The hedging strategies relying on term structure model**

  - overcome by far the MG' strategy

  - are all the more efficient that the model performs well

  - are more efficient when the maturities of the futures contracts are different from each others

Conclusions on hedging strategies

- Empirical studies on hedging strategies are rare
- Little work has been done on the comparison of different strategies
- The frequency of roll-over and the maturity of the futures contracts used for the hedge are arbitrarily chosen

Section 2. Option valuation and investment

- With a term structure model, it is possible to compute a futures price for any expiration date
- Valuation of the net cash flows associated with an investment project
- Framework : real options theory
- Mineral reserves
- Extensively used in the case of petroleum: auctions on undeveloped fields

Real option theory

- Relies on an analogy with financial options
- Aims to identify the optional component included in most investment projects and to evaluate it
- Main advantage: it takes into account the flexibility associated with the project
- Important when irreversibility is associated with the project
- Different categories of real options
- Most investment projects include several options

Real options and mineral reserves

- The analysis framework is quite simple
- Simple valuation models, based on those developed for futures contracts
- Empirical work is rather rare:

  Parameters estimation becomes tricky when the horizon of analysis exceeds the exchange-traded maturities

Pioneer article : Brennan & Schwartz, 1985

- Framework:

  - The resource is homogenous

  - The volume is known

  - Extraction costs are known

  - Interest rates are non-stochastic

  - There is an upper limit to the output rate

  - It is possible to close and reopen the mine in response to market conditions

  - The main source of uncertainty is the price of the commodity

- The dynamic behaviour of the price is represented with the B&S one-factor term structure model
- Several real options associated with the possession of the mine:

  - the option to defer investment

  - the abandonment option

  - the option to shut down the mine temporarily (the option to alter operating scale)

The option to defer investment

- The most extensively used for mineral reserves
- Call option
- Flexibility : possibility to wait for consistent information
- Premium :

  - The holder of the mine gives up immediate cash flows
  - Opportunity cost

- Most important variable:

  Length of the project

- Impact on investment decision:

  Defer or accelerate investment

The abandonment option

- Put option
- Flexibility:
  Definitively renounce to the exploitation of an asset
- Premium: - the holder gives up cash flow
  - loss of skills and competences
- Important factors:
  - high maintenance costs
  - low specificity of the asset

The option to alter operating scale

- Flexibility :
  - increase, reduce or shut down temporarily
- Call and put options
- Important factor :
  - presence of cycles in demand or supply

B&S 1985

- It is never optimal, under uncertainty, to close or abandon the mine
- They show how the option value changes with:

  - the volume of the reserve

  - the initial amount to invest,

  - …

Other studies

- **Real options taken into account**
  - Option to defer investment:
  - Option to abandon
  - Option to alter operating scale

- **Source of uncertainty :**
  - Spot price
  - Convenience yield
  - Long-term price
  - Production rate
  - Technical and geological
  - Environmental constraints

Conclusions on investment

- Empirical work is scarce because of the time horizon in this kind of analysis
- Nowadays, the real option theory is used in order to value oil fields
- The problem with investment projects is that they include, most of the time, several options
- What if these options are not independent ?
- What is the value of a combination of options ?
- What is the value of a combination of options when there are several sources of uncertainty ?

Conclusion on commodity derivative markets

Main characteristics of commodity prices :

- Mean reverting behavior
- Samuelson effect
- High level of volatility
- Volatility rises when commodity prices rise
- Asymmetrical behavior of the basis
- Quality differentials
- Seasonality in the prices and in the basis
- Influence of production costs, transportation costs, storage costs,…
- Jump processes

New trends in research on commodity prices

- Switching regimes
- Options valuation (Asian options)
- Commodity derivatives and IFRS (International Financial Reporting Standards) :
    - Valuation of inventories and mineral reserves
    - Hedging versus speculation
- Convenience yield as a real option
- Price convergence in energy markets
- Commodity as a new class of assets