The term structure of commodity prices

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- The term structure is the relationship, at t, between the spot price and the futures prices for any delivery dates
- Prices curves
- Crude oil (Light Sweet Crude Oil and Brent) : Maximal maturity of 7 years

Section 1. The term structure of commodity prices: an introduction

Section 2. The most famous term structure models

Section 3. Empirical tests on the term structure of commodity prices

Section 1. The term structure of commodity prices: an introduction

1.1. Traditional theories and the term structure

1.2. A long-term extension of the analysis

1.3. Dynamic analysis of the term structure

1.1. Traditional theories and the term structure

- **Normal backwardation theory**
  - Function of transferring the risk between operators
  - Analysis of hedging positions
- **Theory of storage**
  - Motivations for holding stocks
  - Storage costs

- Traditional theories are devoted to short-term analysis
- The theory of storage has a stronger influence
- According to the theory of storage:

  - Three determinants of the futures price:

    - the spot price
    - the convenience yield
    - the interest rate (financing costs)

  - Positive correlation between the spot price and the convenience yield

  - Asymmetry of the basis behavior

1.2. Long-term extension of the analysis

- **Gabillon (1995)**
- **The normal backwardation theory**

  Succession of unbalances on different segments of the curve

  Agents have preferred habitats

- **Theory of storage**

  - Explanatory factors for short-term analysis:
    Production, consumption, stock level, fear of inventory disruptions

  - Explanatory factors for long-term analysis:
    Interest rates, inflation, prices of competing energies

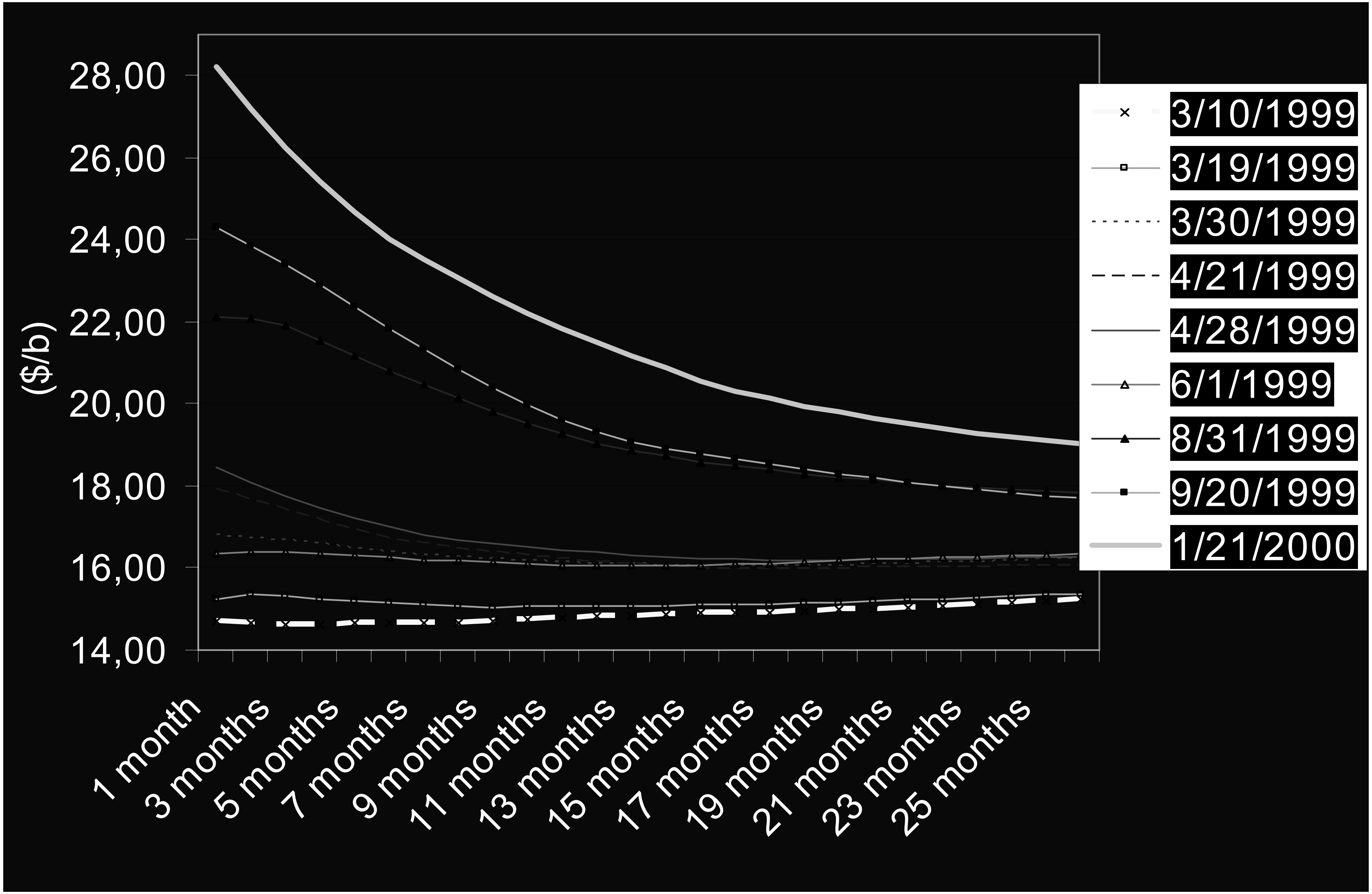
1.3. Dynamic analysis of the term structure

1. Decreasing pattern of volatilities along the prices curve

«Samuelson effect » (1965)

- Empirical validation
- The effect sometimes disappears when stocks are abundant
- Propagation of shocks and storage costs

Fluctuation of prices curves at different dates, WTI



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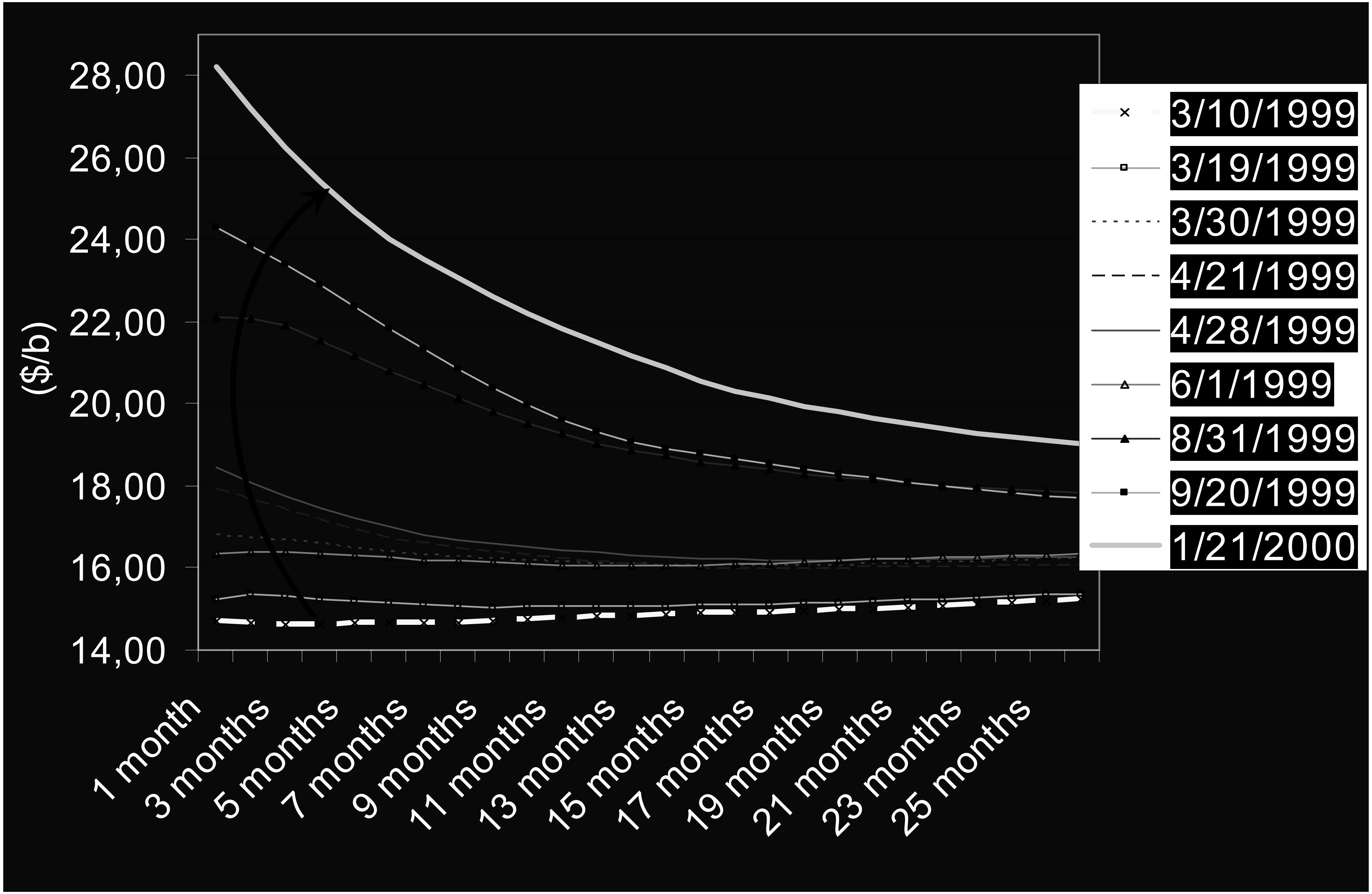

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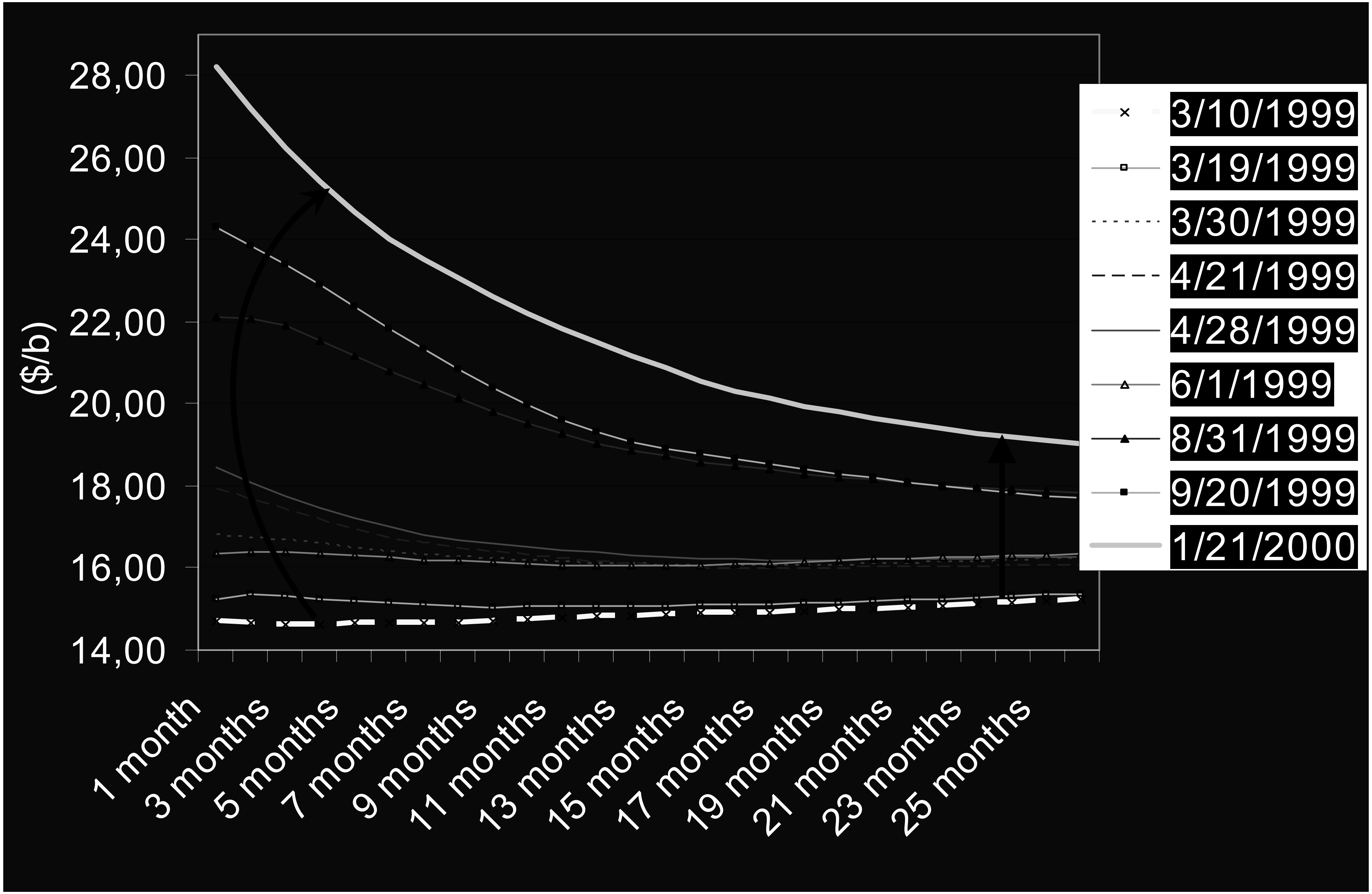
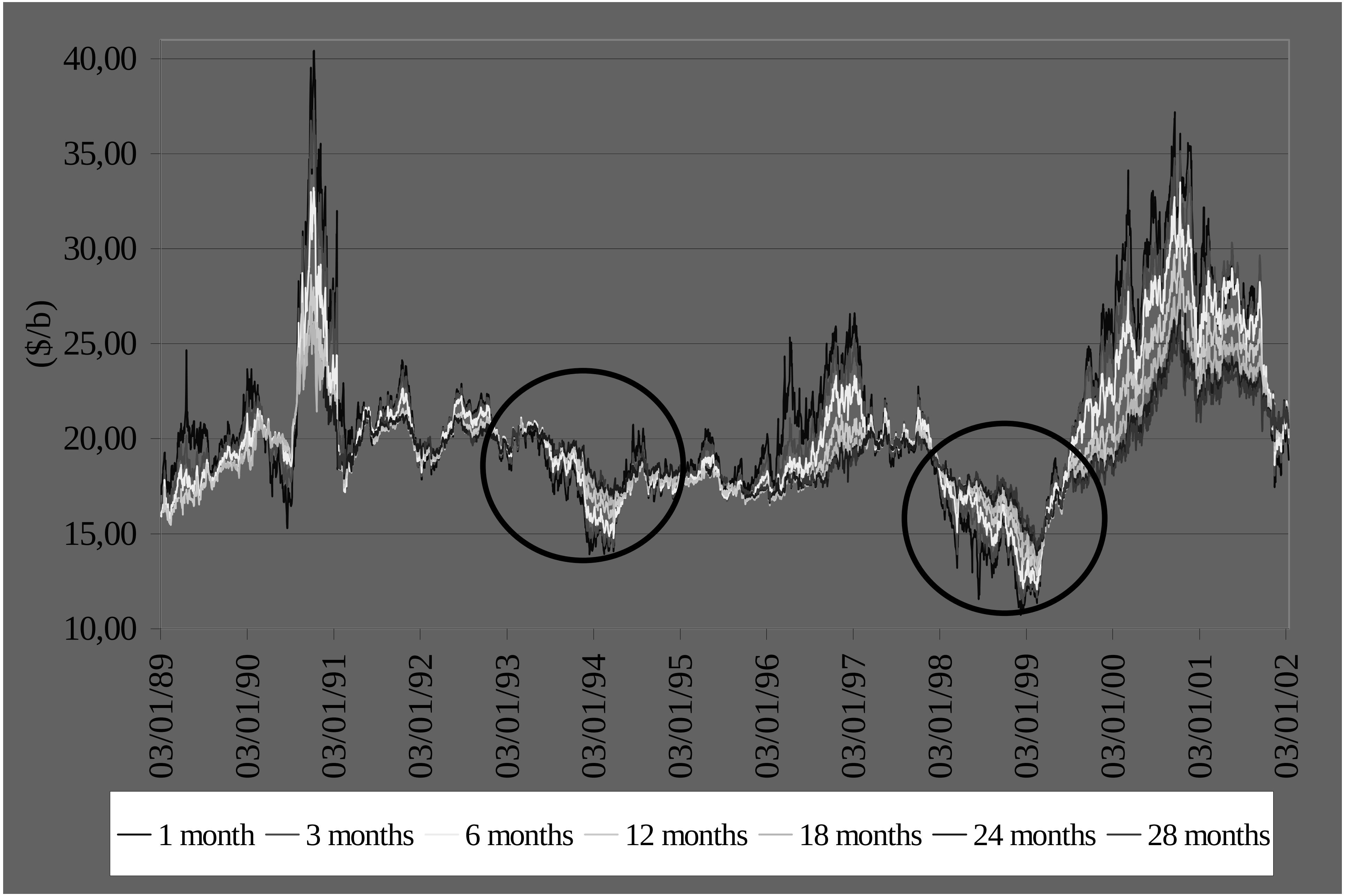


2. Backwardation and the crude oil market

More than 95% of the time

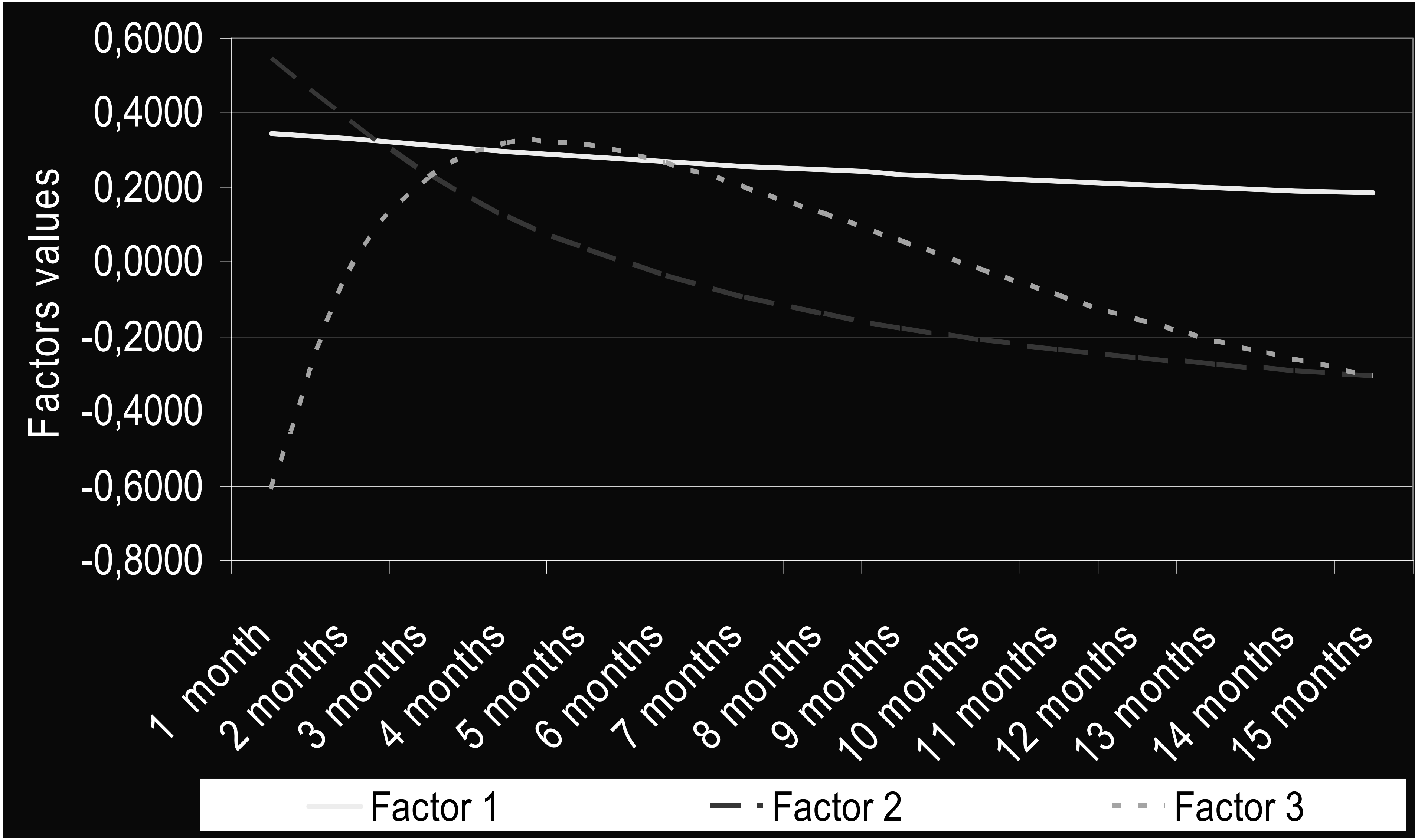
WTI : Futures prices, 1989-2002



3. Movements of prices curves

- Principal component analysis
- Three kind of movements :
  - parallel shift in the curve (level factor)
  - relative shift of the curve (steepness factor)
  - curvature factor

The three factors driving the crude oil prices curve movements 1989-2002



- The dynamics becomes more complex when maturity increases

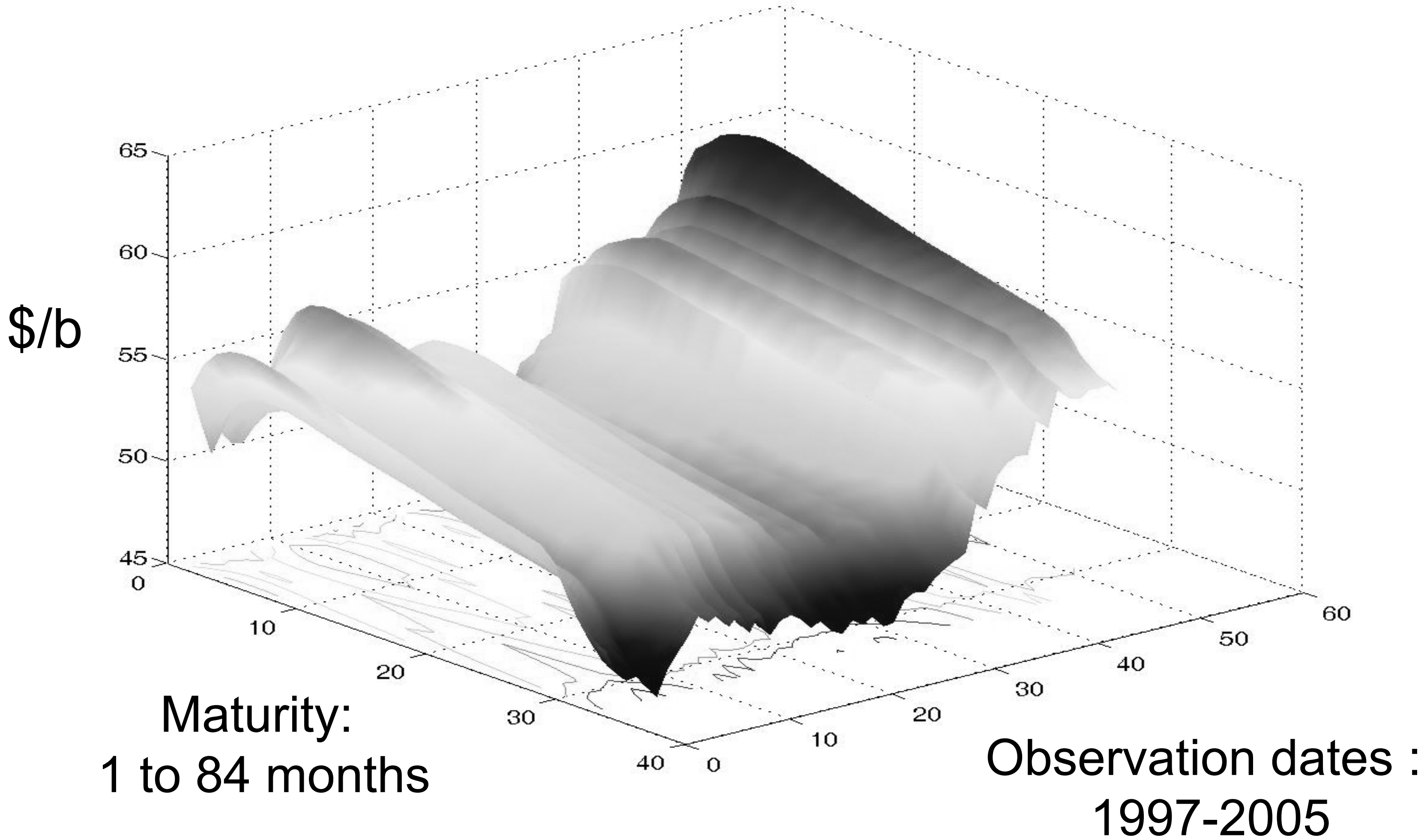
Crude oil market, variability explained by each factor (%), 1999-2002

	1999-2002 (1-15 M)	1999-2002 (1-84 M)
F1	96.15	88.35
F2	3.69	10.81
F3	0.14	0.52

Movements of prices curves : comparison with interest rates

- Frye (1997)
- US Treasury rates with maturities between three months and 30 years
- The first factor accounts for 83.1% of the total variation of the data
- The second accounts for 10%
- The third for 2.8%

Movements of prices curve : illustration



Section 2. The most important term structure models of commodity prices

1. Valuation methods
2. One-factor models
3. Two-factor models
4. Three-factor models

2.1. Contingent claim analysis

Hypotheses:

H1. A derivative asset can be totally specified by a set of factors, namely underlying assets, uncertainty sources, or state variables

H2. The market is free of frictions, taxes or transaction costs

H3. Trading takes place continuously

H4. No short sale constraints

Different steps of the valuation

1. Selection of the state variables and specification of their dynamic behavior
2. Itô’s lemma gives the dynamic behavior of the futures price
3. Arbitrage reasoning and elaboration of a hedge portfolio
4. Fundamental valuation equation and solution of the model

2.2. One-factor models

- A single state variable : the spot price
- Geometric Brownian motion
- Mean reverting behavior
- Other models

Geometric Brownian motion :

Brennan & Schwartz (1985)

$$dS(t) = \mu S(t)dt + \sigma_S S(t)dz$$

- *S* : spot price
- *μ* : drift
- σ : volatility
- *dz* : increment to a standard Brownian motion

Solution :

$$F(S,t,T) = Se^{(r-c)\tau}$$

- *r* : risk free interest rate
- τ = *T* - *t* : maturity of the futures contract

- **<u>Mean reverting behavior (Ornstein-Uhlenbeck process)</u>**

  - Storage behavior of operators facing prices fluctuations on the spot market

  - There is a « normal » level of stocks

  - **Schwartz 1997:**

$$dS = \kappa(\mu - \ln S)Sdt + \sigma Sdz$$

  - S : spot price
  - μ : long-run mean,
  - κ : speed of adjustment,
  - σ : volatility,
  - dz : increment to a standard Brownian motion

$$dX = \kappa(\alpha - X)dt + \sigma dz$$

Solution :

$$F(S,t,T) = \exp\left(\hat{\alpha} + (\ln S - \widehat{\alpha})e^{-\kappa t} + \frac{\sigma^2}{4\kappa}\left(1 - e^{-2\kappa t}\right)\right)$$

Volatility of futures returns:

$$\sigma_F^2 = \sigma^2 e^{-2\kappa t}$$

When τ tend towards infinity:

$$F(S,\infty) = \exp\left(\widehat{\alpha} + \frac{\sigma^2}{4\kappa}\right)$$

Other one factor models : Brennan, 1991

- Convenience yield as a linear function of the spot price :

$$C(S) = c.S$$

- Convenience yield as a non linear function of the spot price

$$C(S) = a + bS + cS^2$$

- Convenience yield and non negativity constraint on stocks

$$C(S) = \max(a, b + cS)$$

2.3. Two-factor models

- **The convenience yield**
  - mean reverting
  - asymmetrical

- **Long term price**

Mean reverting convenience yield

Schwartz 1997

Dynamic of states variables

$$\begin{cases} dS = (\mu - C)Sdt + \sigma_S Sdz_S \\ dC = \left[k\left(\alpha - C\right)\right]dt + \sigma_C dz_C \end{cases}$$

- μ drift of the spot price S,
- σ_i volatility of variable i,
- α *:* long-run mean of the convenience yield C,
- κ : speed of adjustment of the convenience yield,
- *dzi* : Brownian motion .

$$E\left[dz_S \times dz_C\right] = \rho dt$$

Schwartz, 1997

- Convenience yield is a stochastic dividend yield
- Common factors in the term structure : risk premium, return on the underlying asset (stocks, currencies, interest rates)

- Solution of the model

$$F(S,C,t,T) = S(t) \times \exp\left[-C(t)\frac{1-e^{-\kappa\tau}}{\kappa} + B(\tau) \right]$$

$$B(\tau) = \left[\left(r - \hat{\alpha} + \frac{\sigma_C^2}{2\kappa^2} - \frac{\sigma_S \sigma_C \rho}{\kappa} \right) \times \tau \right] + \left[\frac{\sigma_C^2}{4} \times \frac{1-e^{-2\kappa\tau}}{\kappa^3} \right] + \left[\left(\hat{\alpha}\kappa + \sigma_S \sigma_C \rho - \frac{\sigma_C^2}{\kappa} \right) \times \left(\frac{1-e^{-\kappa\tau}}{\kappa^2} \right) \right]$$

$$\hat{\alpha} = \alpha - \left(\lambda / \kappa \right)$$

- r : risk free interest rate
- λ : market price of convenience yield risk,
- $\tau = T - t$: maturity of the futures contract

Volatility of futures prices

$$\sigma_F^2(\tau) = \sigma_S^2 + \sigma_C^2\left(\frac{1-e^{-\kappa\tau}}{\kappa}\right)^2 - \left[2\times\frac{1-e^{-\kappa\tau}}{\kappa}\times\rho\sigma_S\sigma_C\right]$$

When τ tend towards infinity:

$$\lim_{\tau\to\infty}\sigma_F^2 = \sigma_S^2 + \frac{\sigma_C^2}{\kappa^2} - \frac{2\rho\sigma_S\sigma_C}{\kappa}$$

Asymmetrical convenience yield

- Convenience yield as a real option
- Introduction of an asymmetry in the term structure model

The long-term price

- Gabillon, 1992
- Short-term/ Long-term model: Schwartz & Smith, 2000
- Spot price is a function of two stochastic variables:

$$\ln(S_t) = \chi_t + \xi_t$$

χ_t : short-term deviations

ξ_t : equilibrium price level

$$\begin{cases} d\chi_t = -\kappa\chi_t dt + \sigma_\chi dz_\chi \\ d\xi_t = \mu dt + \sigma_\xi dz_\xi \end{cases}$$

- Introduction of the Samuelson effect
- Avoid the critiques addressed to the convenience yield
- In concordance with works on long memory processes
- Is the long-term price stochastic ?

Seasonality

- In the commodity prices

- In the convenience yield

2.4. Three-factor models

- Interest rates

  $\longrightarrow$ forward $\neq$ futures

- Growth rate of the equilibrium price
- Long-term price
- Volatility

Arbitrage between reality and simplicity

Cortazar & Schwartz (2003)

- Three state variables:
  - Spot price
  - Convenience yield
  - Long-term price
- Dynamics of the state variables:

$$\begin{cases} dS = (v - y)Sdt + \sigma_1 Sdz_1 \\ dy = -\kappa y dt + \sigma_2 dz_2 \\ dv = a(\bar{v} - v)dt + \sigma_3 dz_3 \end{cases}$$

$$dz_1 dz_2 = \rho_{12} dt \qquad dz_1 dz_3 = \rho_{13} dt \qquad dz_2 dz_3 = \rho_{23} dt$$

Solution of the model

$$F(S,y,v,t,T)=S(t)\times\exp\left(-y(t)H(\kappa,\tau)+v(t)H(a,\tau)+\varphi(\tau)\right)$$

$$H(i,\tau)=\frac{1-e^{-i\tau}}{i}$$

$$\mu=\bar{\nu}-(\lambda_3+\lambda_2+\lambda_1)$$

$$\varphi(\tau)=\mu\tau+\frac{1}{2}\sigma_3^2\left[\frac{\tau-H(a,\tau)}{a^2}-\frac{H(a,\tau)^2}{2a}\right]+\frac{1}{2}\sigma_2^2\left[\frac{\tau-H(\kappa,\tau)}{\kappa^2}-\frac{H(\kappa,\tau)^2}{2\kappa}\right]$$

$$+\frac{\sigma_1\sigma_3\rho_{13}}{a}\left(\tau-H(a,\tau)\right)-\frac{\sigma_1\sigma_2\rho_{12}}{\kappa}\left(\tau-H(\kappa,\tau)\right)$$

$$-\frac{\sigma_3\sigma_2\rho_{23}}{a+\kappa}\left[\tau\left(\frac{1}{a}+\frac{1}{\kappa}\right)-\frac{1}{\kappa}H(\kappa,\tau)-\frac{1}{a}H(a,\tau)-H(a,\tau)H(\kappa,\tau)\right]$$

Volatility of futures returns

$$\sigma_F^2(\tau)=\sigma_1^2+\sigma_2^2\frac{\left(1-e^{-\kappa\tau}\right)^2}{\kappa^2}+\sigma_3^2\frac{\left(1-e^{-a\tau}\right)^2}{a^2}-2\sigma_1\sigma_2\rho_{12}\frac{\left(1-e^{-\kappa\tau}\right)}{\kappa}+2\sigma_1\sigma_3\rho_{13}\frac{\left(1-e^{-a\tau}\right)}{a}$$
$$-2\sigma_2\sigma_3\rho_{23}\frac{\left(1-e^{-\kappa\tau}\right)\left(1-e^{-a\tau}\right)}{a\kappa}$$

When τ tend towards infinity:

$$\sigma_F^2(\tau\to\infty)=\sigma_1^2+\frac{\sigma_2^2}{\kappa^2}+\frac{\sigma_3^2}{a^2}-\frac{2\sigma_1\sigma_2\rho_{12}}{\kappa}+\frac{2\sigma_1\sigma_3\rho_{13}}{a}-\frac{2\sigma_2\sigma_3\rho_{23}}{a\kappa}$$

Term structure models : conclusion

- **Partial equilibrium models**

  The choice of state variable is somehow arbitrary

  Theory of storage

  Samuelson effect
- Two-factor models: Relative importance of the convenience yield and of the long term price ?
- Term structure of volatilities ?
- Probabilistic approach ?
- General equilibrium model ?

Section 3. Empirical validation of term structure models

3.1. Simulations

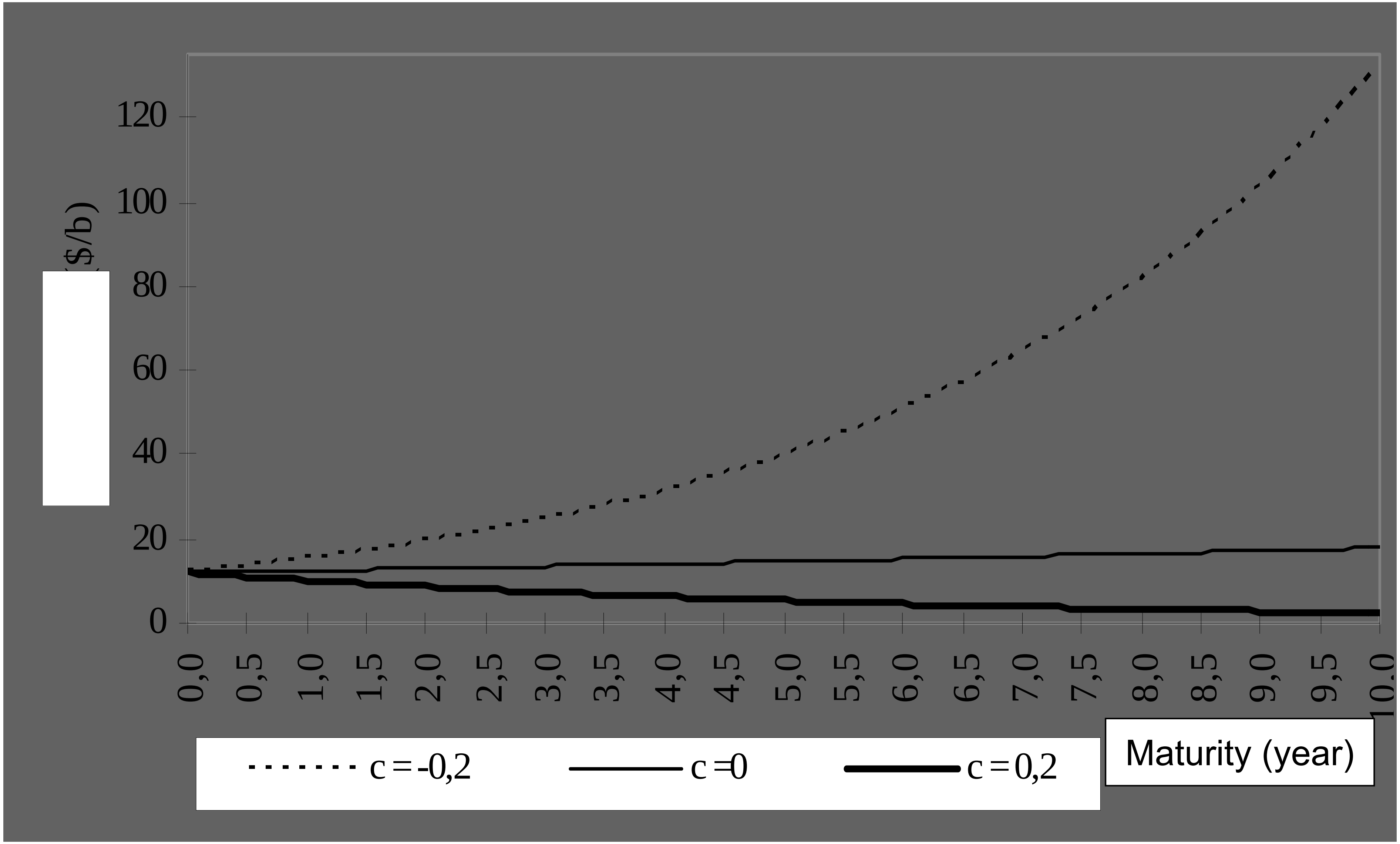
3.2. Parameters estimations

3.3. Model performances

3.1. Simulations

- Brennan & Schwartz, 1985
- Schwartz, 1997

Brennan & Schwartz model
Impact of a variation in the convenience yield



- Prices curves are monotonically decreasing, monotonically increasing or flat
- The relative level of the two parameters (interest rate r and convenience yield c) determine the whole shape of prices curve
- **Growth rate of the futures price :**

$$\frac{1}{F} \times \frac{\delta F}{\delta \tau} = r - c$$

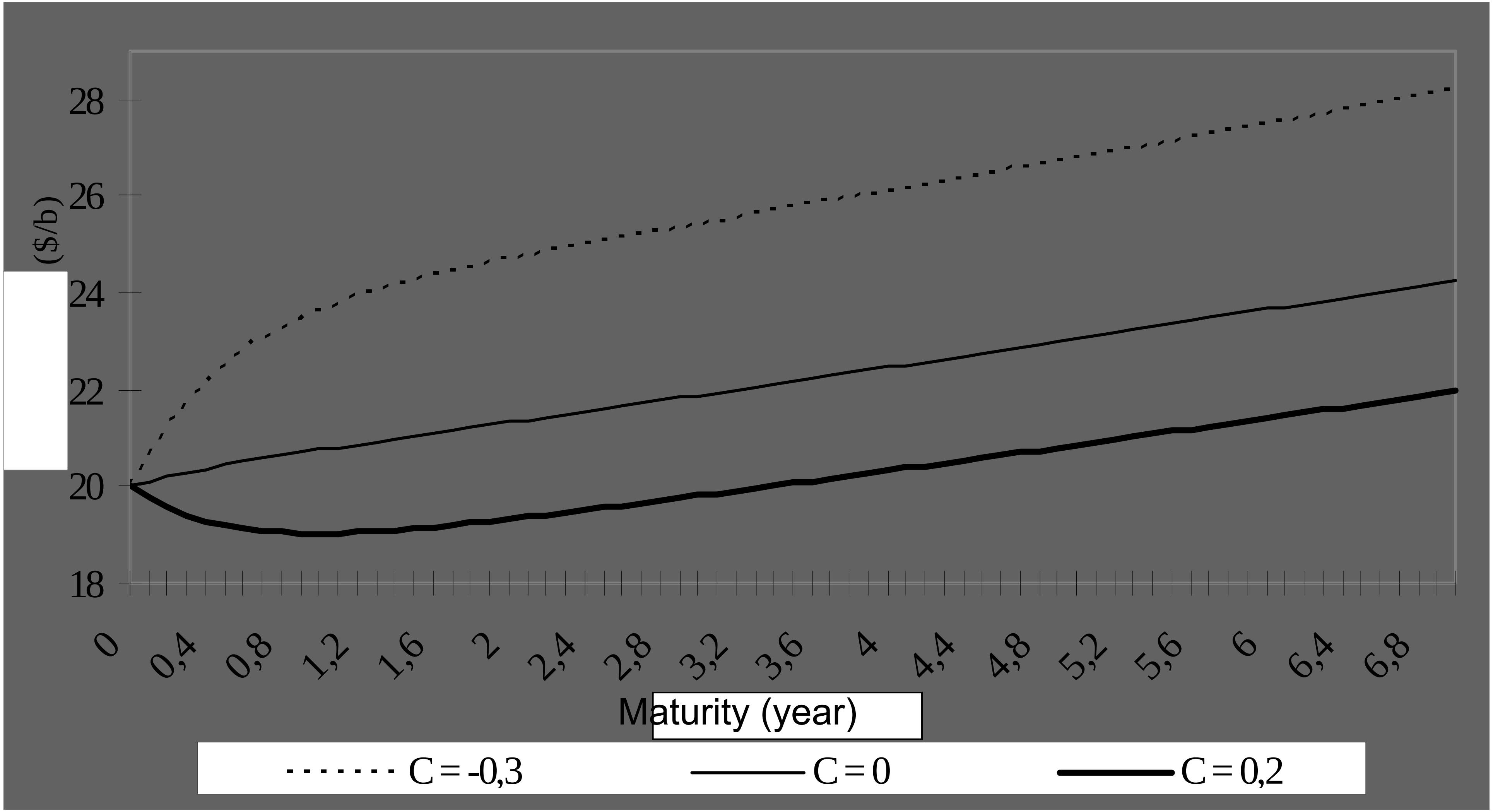
r > c $\longrightarrow$ contango (F>S)

r < c $\longrightarrow$ backwardation (S>F)

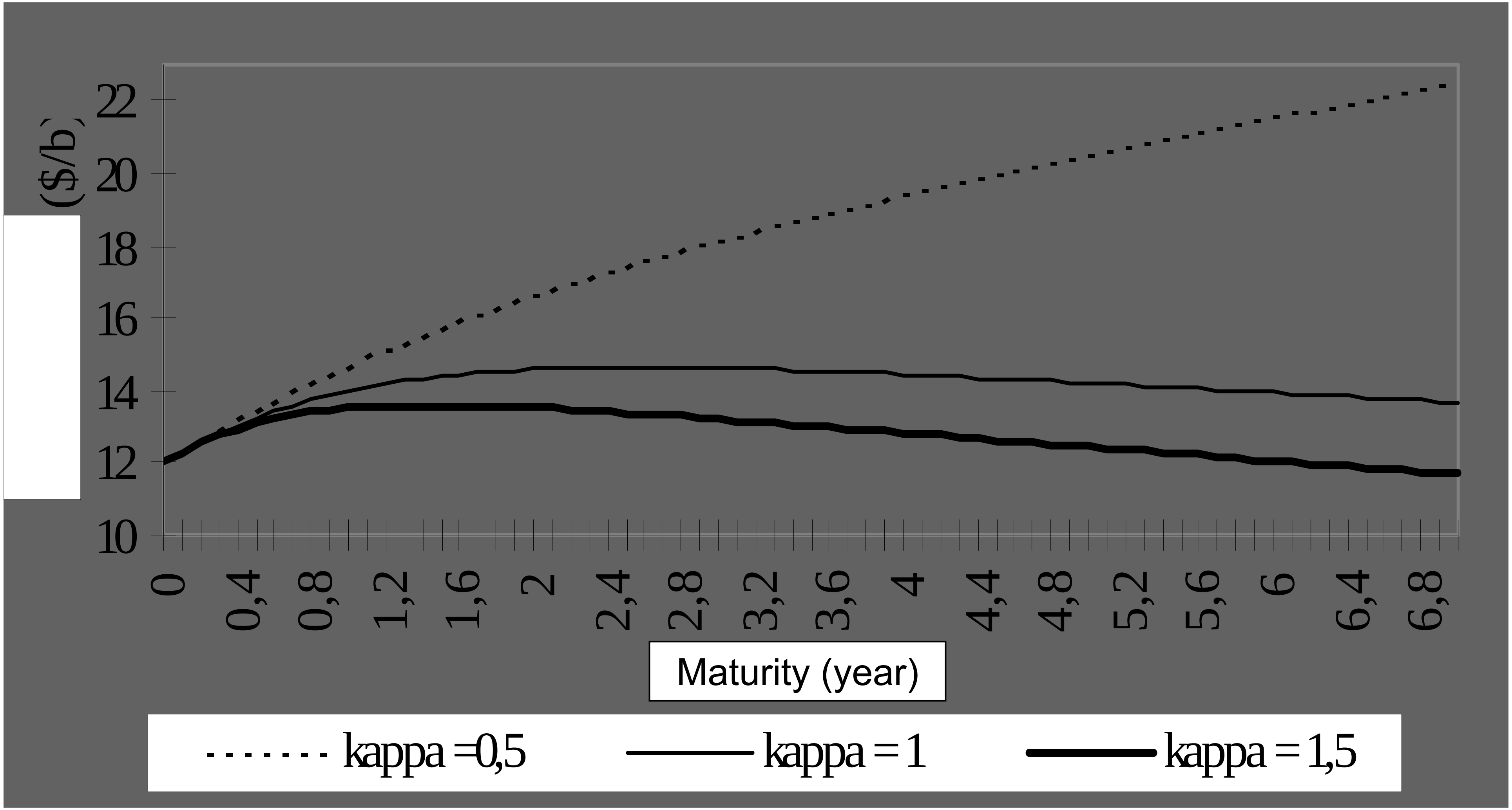
- The values of futures prices can reach a level without real economic significance
- The convenience yield is supposed to be constant
- The behavior of operators in the physical market (their reaction to prices fluctuations) is not taken into account
- **The Samuelson effect is ignored**
  Volatility of the futures prices returns :

$$\frac{dF}{F} = \sigma_S dz$$

Schwartz model,
Impact of a variation in the convenience yield



Schwartz model,
Impact of a variation in the speed of adjustment



- **The introduction of a second state variable allows for various shapes of prices curves**

- For the nearest expiration dates, the shape of the prices curve depends strongly on the values of C and kappa

- When maturity tends toward infinity, the volatility of the futures price tends toward a fixed value

- The volatility of futures prices decreases with the maturity of the futures contract.

3.2. Parameters estimation

- **Non-observable state variables** :

  - **spot price**
    - physical markets are geographically dispersed,
    - transactions are not standardized
    - reporting mechanism
  - **convenience yield** : non traded asset
  - **long term price** : non traded asset

- **Kalman filtering**

Kalman filters

- Allow the reconstitution of series of non observable variables
- Provide a way to estimate the parameters
- Different versions of Kalman filters :

  - Linear models : simple Kalman filter

  - Non linear models : extended Kalman filter

  - Non Gaussian models : particle filters

- State-space model characterized by:
  - Transition equation
  - Measurement equation
- Iteration procedure, with three steps :
  - Prediction
  - Innovation
  - Updating
- Parameters estimation
- Reconstitution of series of non observable variables

The simple Kalman filter

- State-space model, characterized by two equations
  - Transition equation
  - Measurement equation
- **Transition equation:**

$$\alpha_{t/t-1} = T\alpha_{t-1} + c + R\eta_t$$

- α_t : m-dimensional vector of non-observable variables at t (state vector)
- T : (m $\times$ m) matrix
- c : m-dimensional vector
- R : (m $\times$ m)

- **Measurement equation:**

$$y_{t/t-1} = Z\alpha_{t/t-1} + d + \varepsilon_t$$

- $y_{t/t-1}$: N-dimensional temporal series
- Z : (N×m) matrix
- *d* : m-dimensional vector
- η_t and ε_t are white noises whose dimensions are respectively m and N. They are supposed to be normally distributed

$$E[\eta_t] = 0 \qquad Var[\eta_t] = Q$$

$$E[\varepsilon_t] = 0 \qquad Var[\varepsilon_t] = H$$

- Initial value of the system is supposed to be normal
- Mean and variance:

$$E[\alpha_0] = \tilde{\alpha}_0$$

$$Var[\alpha_0] = P_0$$

- $\tilde{\alpha}_t$ is a non biased estimator of α_t, conditionally on the information available at t:

$$E_t[\alpha_t - \tilde{\alpha}_t] = 0$$

- Covariance matrix P_t :

$$P_t = E_t[(\tilde{\alpha}_t - \alpha_t)(\tilde{\alpha}_t - \alpha_t)']$$

Iteration procedure

- Three steps:

  - prediction
  - innovation
  - updating

- **Prediction:**

$$\begin{cases} \tilde{\alpha}_{t/t-1} = T\tilde{\alpha}_{t-1} + c \\ P_{t/t-1} = T\, P_{t-1}\, T' + R\, Q\, R' \end{cases}$$

$\tilde{\alpha}_{t/t-1}$ and $P_{t/t-1}$ are the best estimators of $\tilde{\alpha}_{t-1}$ and P_{t-1}, conditionally on the information available at (t-1).

- **Innovation :**

$$\begin{cases} \tilde{y}_{t/t-1} = Z\tilde{\alpha}_{t/t-1} + d \\ v_t = y_t - \tilde{y}_{t/t-1} \\ F_t = ZP_{t/t-1}Z' + H \end{cases}$$

$\tilde{y}_{t/t-1}$: estimator of the observation y_t conditionally on the information available at (t-1)

v_t : innovation process

F_t *:* covariance matrix

- **Updating :**

$$\begin{cases} \tilde{\alpha}_t = \tilde{\alpha}_{t/t-1} + P_{t/t-1} Z' F_t^{-1} v_t \\ P_t = (I - P_{t/t-1} Z' F_t^{-1} Z) P_{t/t-1} \end{cases}$$

Applying the simple Kalman filter to Schwartz model (1997)

- The simple filter is suited for linear models :

$$\ln\left(F(S,C,t,T)\right)=\ln\left(S(t)\right)-C(t)\times\frac{1-e^{-\kappa\tau}}{\kappa}+B\left(\tau\right)$$

- Letting *G* = *ln(S)*, we also have:

$$\begin{cases} dG=(\mu-C-\frac{1}{2}\sigma_S^2)dt+\sigma_S dz_S \\ dC=\left[k\left(\alpha-C\right)\right]dt+\sigma_C dz_C \end{cases}$$

From Schwartz model to a state-space model:

- Transition equation (dynamics of the state variables)

$$\begin{bmatrix} \tilde{G}_{t/t-1} \\ \tilde{C}_{t/t-1} \end{bmatrix} = c + T \times \begin{bmatrix} \tilde{G}_{t-1} \\ \tilde{C}_{t-1} \end{bmatrix} + R\eta_t \qquad \text{t= 1, ... NT}$$

- N : number of maturities used for the estimation
- Δt : period between 2 observation dates
- *R :* identity matrix, (2×2)
- η_t : errors that are uncorrelated with the previous values of the state variables, and have no serial correlation : $E[\eta_t] = 0$

$$Q = Var[\eta_t] = \begin{bmatrix} \sigma_S^2 \Delta t & \rho\sigma_S\sigma_C\Delta t \\ \rho\sigma_S\sigma_C\Delta t & \sigma_C^2 \Delta t \end{bmatrix}$$

$$c = \begin{bmatrix} \left(\mu - \frac{1}{2}\sigma_S^2\right)\Delta t \\ \kappa\alpha\Delta t \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & -\Delta t \\ 0 & 1 - \kappa\Delta t \end{bmatrix}$$

Measurement equation (solution of the model):

$$\tilde{y}_{t/t-1} = d + Z \times \begin{bmatrix} \tilde{G}_{t/t-1} \\ \tilde{C}_{t/t-1} \end{bmatrix} + \varepsilon_t \qquad \text{t= 1, ... NT}$$

- The i[th] line of the N dimensional vector of the observable variables is $\ln\left(\tilde{F}(\tau_i)\right)$, with i = 1,..,N,
- *d* = [*B(*τ_i*)*] is the i[th] line of the *d* vector*,* with i = 1,..., N
- $Z = [1 \ , -H_i]$ is the i[th] line of the *Z* matrix, which is (N×2), with i = 1,...,N and where:

$$H_i = \frac{1 - e^{-\kappa \tau_i}}{\kappa}$$

- ε_t is a white noise vector, (N×1), with no serial correlation:

E[ε_t*] = 0* and *H = Var[*ε_t*].* (N × N)

Extended Kalman filter

- **Transition equation:**

$$\alpha_{t/t-1} = T(\alpha_{t-1}) + R(\alpha_{t-1})\eta_t$$

- **Measurement equation:**

$$y_{t/t-1} = Z(\alpha_{t/t-1}) + \varepsilon_t$$

Linearization:

$$\begin{cases} \alpha_{t/t-1} \approx \hat{T}\alpha_{t-1} + \hat{R}\eta_t \\ y_{t/t-1} \approx \hat{Z}\alpha_{t/t-1} + \varepsilon_t \end{cases}$$

$$\hat{Z} = \left.\frac{\delta Z(\alpha_{t/t-1})}{\delta \alpha'_{t/t-1}}\right|_{\alpha_{t/t-1} = \tilde{\alpha}_{t/t-1}} \qquad \hat{T} = \left.\frac{\delta T(\alpha_{t-1})}{\delta \alpha'_{t-1}}\right|_{\alpha_{t-1} = \tilde{\alpha}_{t-1}}$$

$$\hat{R} = R(\tilde{\alpha}_{t-1}) \approx R(\alpha_{t-1})$$

- **Prediction**

$$\begin{cases} \tilde{\alpha}_{t/t-1} = T(\tilde{\alpha}_{t-1}) \\ P_{t/t-1} = \hat{T} P_{t-1} \hat{T}' + \hat{R} Q \hat{R}' \end{cases}$$

- **Innovation**

$$\begin{cases} \tilde{y}_{t/t-1} = Z(\tilde{\alpha}_{t/t-1}) \\ v_t = y_t - \tilde{y}_t \\ F_t = \hat{Z}_t P_{t/t-1} \hat{Z}_t' + H \end{cases}$$

- **Updating**

$$\begin{cases} \tilde{\alpha}_t = \tilde{\alpha}_{t/t-1} + P_{t/t-1} \hat{Z}_t' F_t^{-1} v_t \\ P_t = \left(I - P_{t/t-1} \hat{Z}_t' F_t^{-1} \hat{Z}_t \right) P_{t/t-1} \end{cases}$$

Parameters estimation

- The non-observable variables and the errors are supposed to be normally distributed.
- Maximum likelihood to estimate the parameters
- Compute, at each iteration, the logarithm of the likelihood function for the innovation v_t :

$$\log l(t) = -\left(\frac{n}{2}\right) \times \ln(2\Pi) - \frac{1}{2}\ln(dF_t) - \frac{1}{2} v_t' \times F_t^{-1} \times v_t$$

- Minimization of the log of the likelihood function
- Use the filter and the optimal parameters to reconstitute the non-observable variables and the measure

3.3. Performances of the model

- **Performances criteria**

  Mean Pricing Error :

$$MPE = \frac{1}{N}\sum_{n=1}^{N}\left(\widehat{F}_{\tau,n} - F_{\tau,n}\right)$$

  Root mean-squared error :

$$RMSE = \sqrt{\frac{1}{N}\sum_{n=1}^{N}\left(\widehat{F}_{\tau,n} - F_{\tau,n}\right)^2}$$

Empirical results

- **General features :**

  Parameters change :

  - with the study period
  - with the maturity
  - with initial conditions

- **One-factor models**

  Poor performances

  Non industrial commodities (precious metals)

Sensitivity of the optimal parameters to the initial conditions, Kalman filter Cortazar & Schwartz (2003)

	Case 1	Case 2	Case 3	Case 4	Case 5
σ_1	0.30	0.27	0.27	0.29	0.28
σ_2	0.05	0.15	0.12	0.17	0.05
σ_3	0.09	0.10	0.05	0.05	0.13
ρ_{12}	0.95	0.64	0.94	0.77	0.49
ρ_{13}	-0.71	-0.41	0.62	0.69	-0.71
ρ_{23}	-0.72	0.36	0.46	0.95	0.26
κ_y	0.05	2.43	0.05	0.05	2.22
κ_v	0.05	0.06	0.05	0.05	0.32
μ	-1.84	-0.50	2.42	-3.71	-0.06

- **Two-factor models**

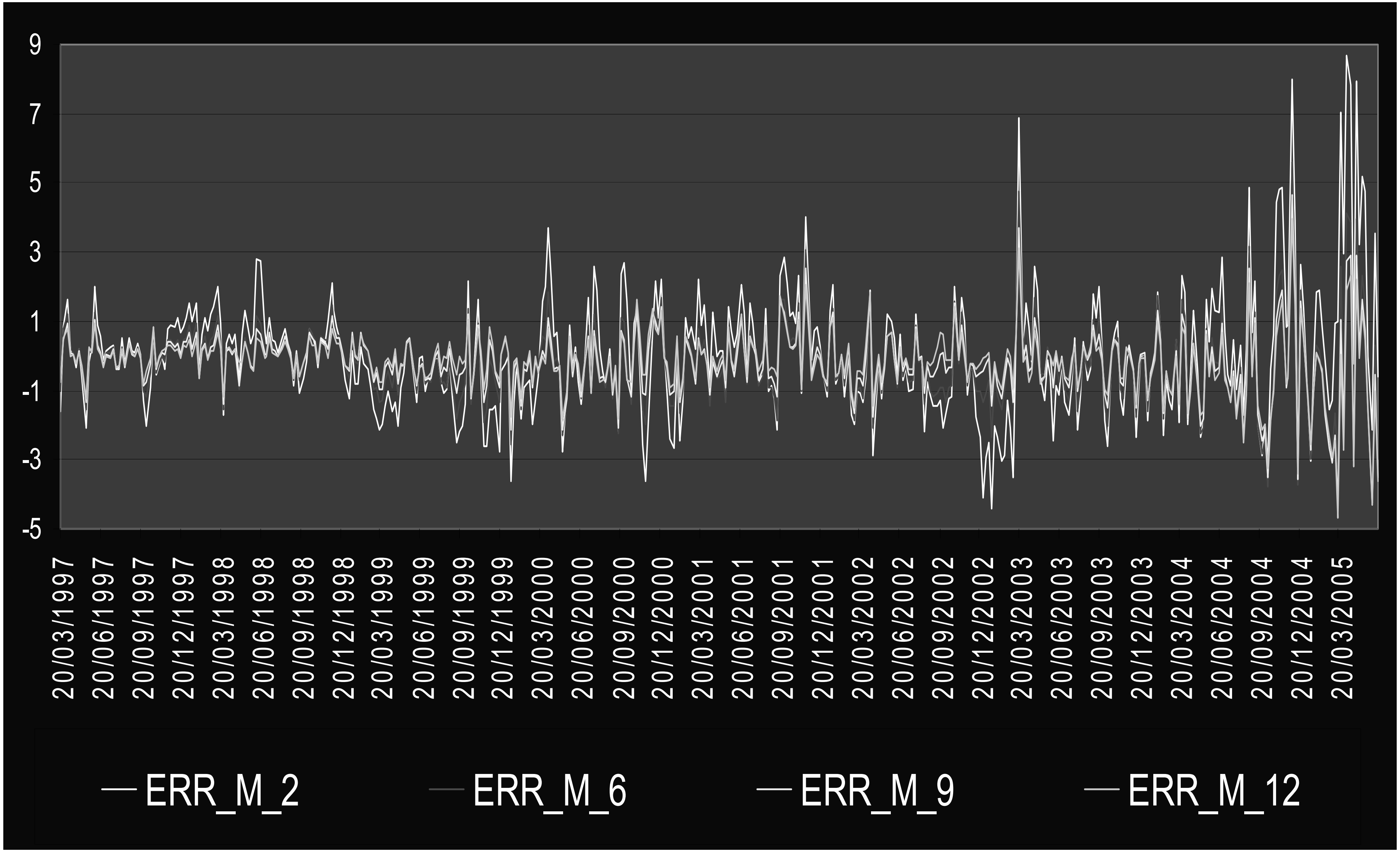
- Convenience yield is mean reverting
- Excellent performances of Schwartz' model (even for long term maturities)
- Performances are improved with an asymmetrical convenience yield

Performances of three models, 1997 - 2005

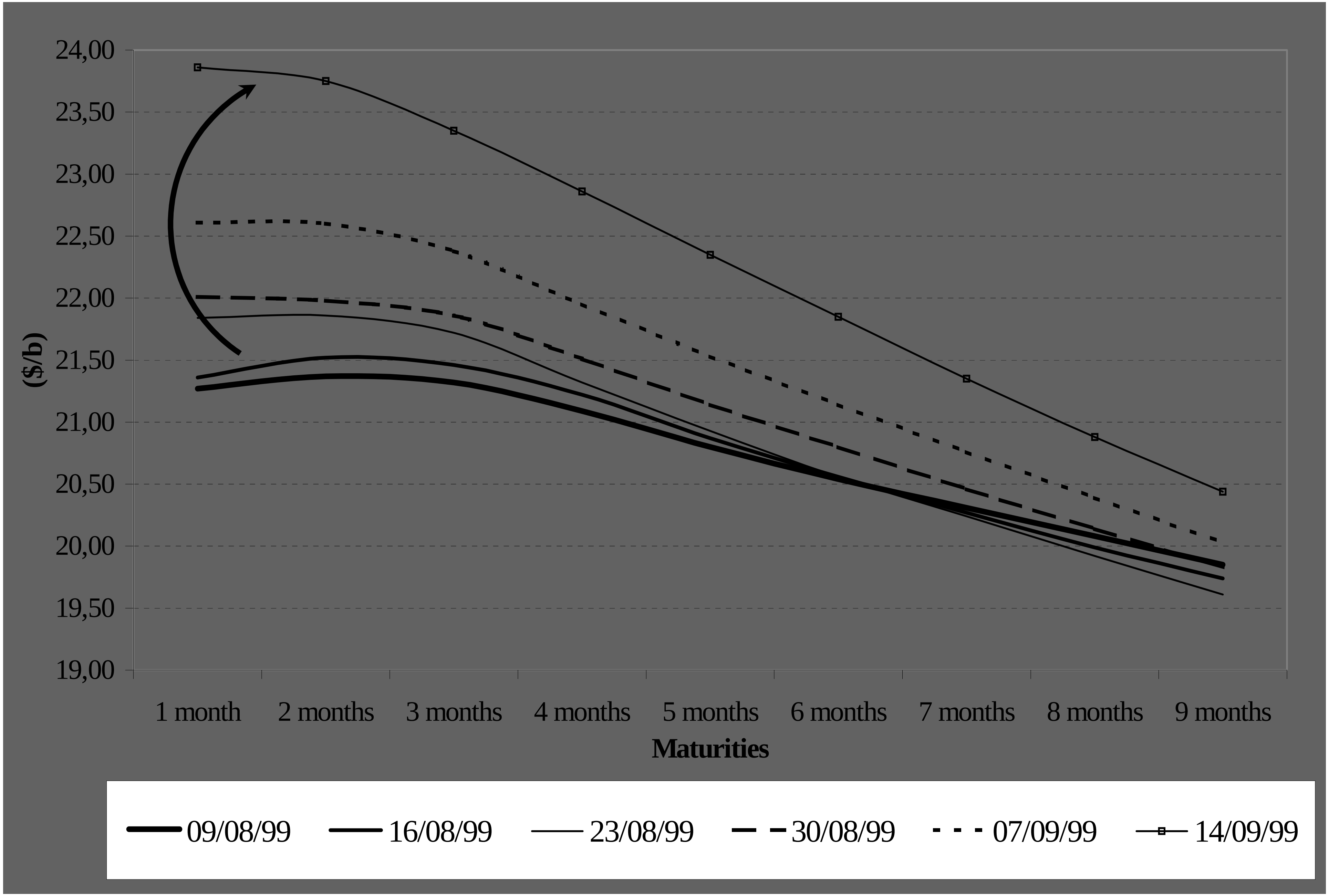
		2M	6M	9M	12M	Mean
1 Factor (Schwartz 97)	MPE	*-0.1532*	*0.3494*	*0.2355*	*0.1663*	***0.1495***
	RMSE	*2.7316*	*1.8427*	*1.1859*	*1.0313*	***1.6979***
2 Factors (Schwartz 97)	MPE	*-0.3069*	*0.1790*	*0.1435*	*0.2161*	***0.0579***
	RMSE	*1.8216*	*1.4312*	*1.1371*	*1.0143*	***1.3510***
3 Factors (Cortazar - Schwartz 03)	MPE	*-0.3401*	*0.2030*	*0.1662*	*0.2064*	***0.0589***
	RMSE	*1.8334*	*1.4346*	*1.1377*	*1.0152*	***1.3552***

Schwartz' two-factor model
Estimated versus observed two-month futures prices

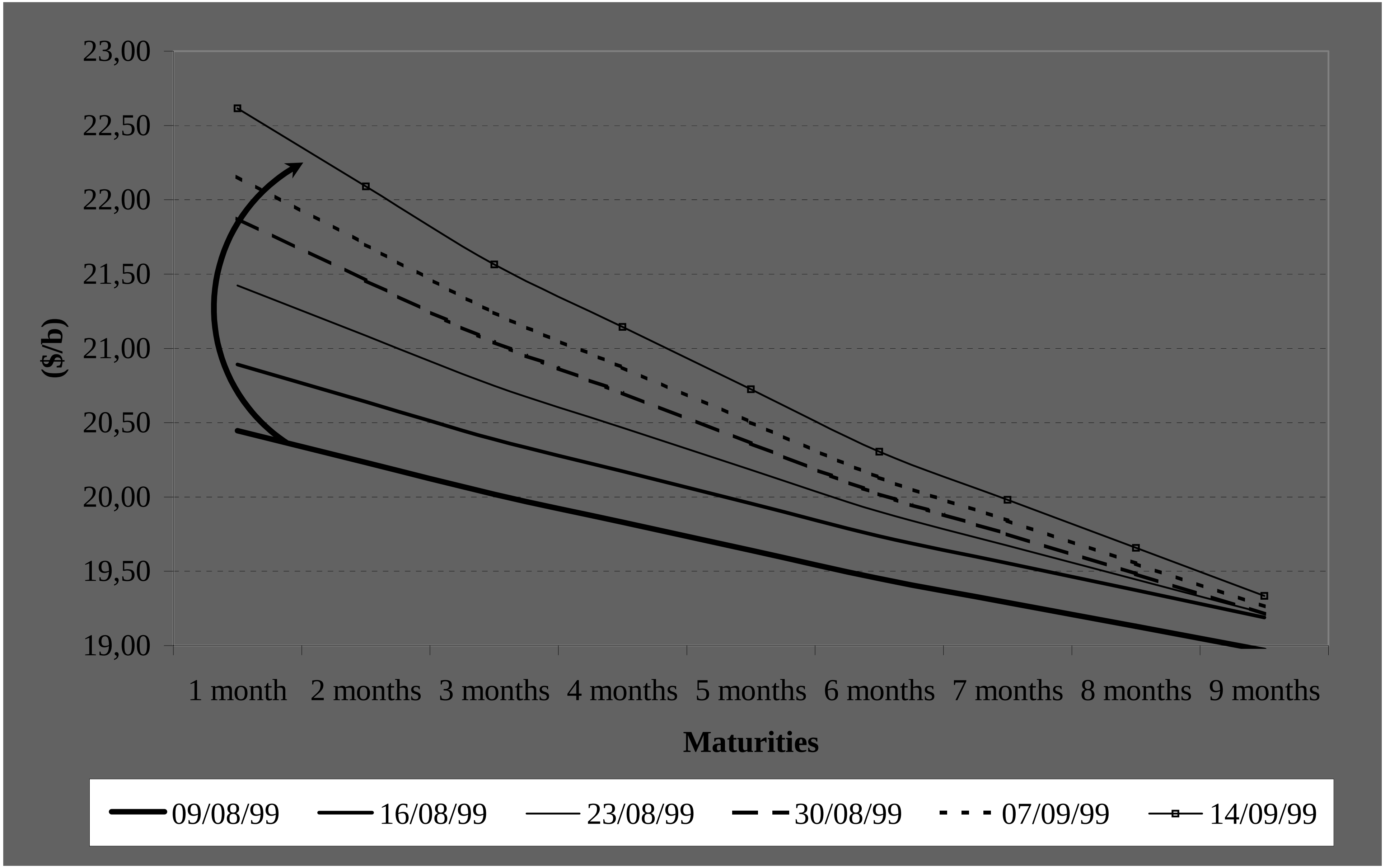
Schwartz' model, Pricing errors



WTI, Observed term structures at different dates



WTI Estimated term structures (with Schwartz model)



Conclusion

- **Common points shared with the term structure of interest rates**

  - Contingent claim analysis

  - Presence of non observable variables