Eighth
Coast Combinatorics Conference

February 24 and 25, 2007
University of Victoria

Room ECS 116

Schedule for Saturday, February 24

9:00 Coffee + goodies at ECS 116

9:30 Rick Brewster, Thompson Rivers University, Solution to the restricted homomorphism conjecture

10:05 Laura Yang, University of Alberta, Postnikov's Identity and its Generalizations

10:40 Aaron Williams, UVic, Generating multi-set permutations in a cool way

11:15 Luis Goddyn, SFU, Vanishing Bases in Projective Geometries

11:45 - 1:30 LUNCH on your own. Suggestions? Look at the page following Sunday's schedule.

1:30 Wendy Myrvold, UVic, Ally and Adversary Reconstruction Numbers

2:05 Tom Brown, SFU, On colorings of the factors of a word

2:40 Gara Pruesse, Malaspina, A new proof for Knuth's old sum

3:05 **Break** on your own. Finnerty's cafe, under the Bookstore, is open

3:30 Moshe Rosenfeld, U. Washington, Famous and lesser known problems in "elementary" combinatorial geometry

4:30 Mohammad Ghebleh, SFU, Computing the circular chromatic number

That's all for today. See you at Swans around 7:30?

Schedule for Sunday, February 25

9:00 Coffee + goodies at ECS 116

9:30 Joe Peters, SFU, Constructing Incremental Sequences in Graphs

10:05 Frank Ruskey, UVic, Hamming distance from an irreducible polynomial

10: 40 Art Finbow, SMU, Remarks on Well-Covered Graphs of Girth 4

11:15 Petr Lisonek, SFU, Planar Eulerian triangulations are equivalent to spherical latin bitrades

11:45 - 1:30 Lunch on your own. Suggestions on the next page.

1:30 Mark Weston, UVic, Symmetries of Venn Diagrams Embedded on the Sphere

2:05 Brett Stevens, Carleton, Locating and avoiding errors in software testing

2:40 Stephen Finbow, StFX, Open-Open Irredundance

3:15 Gary MacGillivray, UVic, Fractional maximal independent sets in graphs

Thanks for coming!

Useful Information

Internet access: go to the main Library and use one of the Public Access teminals (They are Mac's, and no password is needed).

Places to eat on campus

- The Main Servery in the Cadboro Commons (the door is on the left side of the building when your back is to campus, once inside you need to go upstairs). Open 7-7 daily, except for 2 hours in the mid-afternoon.
- Finnerty Express, under the Bookstore, has good coffee and snacks
- The Student Union Building has a number of food kiosks and cafes some of which may be open on Saturday

Food near campus.

There are 4 directions to go:

- Down Sinclair Road to Cadboro Bay Village (20 mins walking). There is Starbucks, two restaurants and a pub, plus other services.
- West (= left if coming from campus) along McKenzie to Shelbourne (25 mins walking). There is a Tim Hortons, a Boston Pizza, a family restaurant, and other services.
- West down Cedar Hill X road to Shelbourne (15 mins walking). More specifiically, go out the main entrance to UVic and turn right. There is Tim Hortons and fast food like McDonals, plus a pub , a Chinese Restaurant, a muffin and soup place, a family restaurant (up the hill in the mall behind McDonalds) and other services and restaurants in the Shelbourne Plaza mall.
- South along Henderson Rd. (out the main entrance) and down Foul Bay Rd. (10 mins drive, not walkable). There are services at Fort St. and Foul Bay Rd. including a grocery store, restaurants, a bank, and cafs. Three minutes farther on, after turning left at Oak Bay Avenue, you come to Oak Bay Village which has many restaurants and services.

Abstracts

In alphabetical order by speaker surname

Solution to the restricted homomorphism conjecture
Richard Brewster, Thompson Rivers University

Let H be a fixed graph. Given a graph G, testing the existence of a homomorphism of G to H is a notoriously difficult problem. A natural question to ask is whether the problem becomes easier if allowed instances, G, are restricted in some natural way. For example, if G maps to C_5, then G must be 3-colourable. Is testing the existence of a homomorphism to C_5 easy for 3-colourable graphs?

For fixed H and Y, the *Restricted Homomorphism Problem* takes as input G and a homomorphism $g : G \to Y$. The question is does G admit a homomorphism to H? Hell and Nešetřil conjectured that this problem is polynomial if (i) H contains a loop, (ii) H is bipartite, or (iii) $Y \to H$; otherwise the problem is NP-complete. We prove this conjecture.

This joint work with Timothy Graves.

On colourings of the factors of a word
Tom Brown, Simon Fraser University

We prove some Ramsey-like theorems on colorings of the factors of finite and infinite words over an arbitrary alphabet. The theorems here are related to results proved by M. -P. Schtzenberger in 1966. Here is a typical theorem: *Let the factors of an infinite word s be finitely colored. Then there is a set U of factors, $U = \{u_1, u_2, u_3, \ldots\}$, such that (1) $s = tu_1u_2u_3\ldots$, and (2) for $1 \le i < j$ all of the factors $u_iu_{i+1}u_{i+2}\ldots u_{j-1}$ (including the factors $u_i, i \ge 1$) have different lengths and have the same color.*

AMS Mathematics Subject Classification: 05D10
Keywords: factor, word, coloring, Ramseys theorem

Remarks on Well-Covered Graphs of Girth 4

Art Finbow, Saint Marys University

A graph G is said to be well-covered if every maximal independent set of vertices has the same cardinality. The problem of determining if a graph is well covered is known to be co-NP-complete. I will speak about some progress in the study of well-covered graphs of girth 4. In one project (joint work with B. Hartnell) we are focusing on the subclass of W_2 graphs i.e. well-covered graphs in which the removal of any vertex leaves a well-covered graph with the same independence number. In another project (joint work with C. Whitehead) we are focusing on the subclass of type I graphs i.e. well-covered graphs that contain a vertex with degree equal to the independence number, and whose removal leaves a non well-covered graph.

Open-Open Irredundance

Stephen Finbow, St. Francis Xavier University

A vertex set X is called OO-irredundant if for every $x \in X$, the set $N(x) - N(X - x) \neq \emptyset$. The concept of OO-irredundance was first studied by Farley and Schacham (1983) as a geralisation of irredundance. Our discussion of the concept will focus on results towards a lower bound on the size of a maximal OO-irredundant set in terms of order and maximum degree. Analogous best possible bounds are known for irredundance, CO-irredundance and OC-irredundance.

Computing the circular chromatic number

Mohammad Ghebleh, Simon Fraser University

We present a greedy algorithm for circular colouring. Similar to the odinary colouring, this greedy algorithm can be integrated into meta-heuristic methods for circular colouring. We present an implementation of a TABU search for circular colouring.

Vanishing Bases in Projective Geometries
Luis Goddyn, Simon Fraser University

Here is a curious fact: *If each point of a finite projective plane (of any order) is assigned an integer weight, then there exist three non-collinear points whose weights sum to a multiple of three.*

This fact might generalize as follows: *If we assign integer weights to the points of a finite projective geometry* $PG(n-1,q)$ *of dimension* $n-1$ *(of any order* q*), then there exists a basis (*n *points in general position) whose weights sum to 0 modulo* n*.*

We prove this (and more) when n is either a prime power or the product of two primes. This is joint work with Matt DeVos and Bojan Mohar.

Planar Eulerian triangulations are equivalent to spherical latin bitrades. Petr Lisonek, Simon Fraser University

Given a pair of latin squares, we may remove from both squares those cells that contain the same symbol in corresponding positions. The resulting pair of partial latin squares is called a latin bitrade. The size of a latin bitrade is the number of filled cells in either of the two partial latin squares that constitute the bitrade.

There is a natural way to define the genus of a latin bitrade; the bitrades of genus 0 are called spherical. We construct a bijection between the isomorphism classes of planar Eulerian triangulations on v vertices and the main classes of spherical latin bitrades of size $v-2$. This results in a very fast algorithm for generating all non-isomorphic spherical latin bitrades of a given size.

This is joint work with Nick Cavenagh (University of New South Wales).

Fractional maximal independent sets in graphs
Gary MacGillivray, University of Victoria

In their book on domination in graphs, Haynes *et al.* asked if it is possible to define a fractional independent set in a graph as a function $f : V \to [0,1]$ such that:

1. the characteristic function of an independent set is a fractional independent set, and
2. there is a concept of maximality so that:
   (a) the characteristic function of a maximal independent set is a *maximal fractional independent set* (MFIS), and
   (b) every MFIS is a minimal fractional dominating set.

Such a definition was provided by K. Reji Kumar in his Ph.D. thesis.

Motivated by the question of the possible *aggregate value* of a MFIS (the sum of $f(x)$ over all vertices $x \in V$), we explore the question of when a convex combination of MFISs is also a MFIS. The graphs for which there exists *universal MFIS* (a MFIS f such that any convex combination of f and a MFIS g is a MFIS) are characterised, as are the graphs for which the set of MFISs is convex.

This is joint work with K. Reji Kumar and S. Arumugam.

Ally and Adversary Reconstruction Numbers
Wendy Myrvold, University of Victoria

The *deck* of a graph G is its multiset of vertex-deleted subgraphs $G - v$ for all vertices v. A graph G is *reconstructible* from a subset S of its deck if every graph H whose deck contains S is isomorphic to G. One of the major unsolved problems in graph theory is to prove the Reconstruction Conjecture which states that all graphs on at least three vertices are reconstructible from the complete deck.

The *ally-reconstruction number* of a graph is the cardinality of a smallest subset of the deck from which a graph is reconstructible. The *adversary-reconstruction number* is the cardinality of a maximum subset S of the deck from which a graph G is reconstructible with the additional property that G is not reconstructible from any subset of S. Harary and Plantholt initiated the study of reconstruction numbers of a graph. This talk surveys work which has been done on reconstruction numbers focussing on the questions which this work leaves open.

Joint work with: K. J. Asciak, M. A. Francalanza, J. Lauri, University of Malta; E-mail: josef.lauri@um.edu.mt, wendym@cs.UVic.ca

Keywords: Graph reconstruction, vertex-deleted subgraphs, isomorphism

Constructing Incremental Sequences in Graphs
Joe Peters, Simon Fraser University

Given a weighted graph $G = (V, E, w)$, we investigate the problem of constructing a sequence of $n = |V|$ subsets of vertices $M_1, ..., M_n$ (called groups) with small diameters, where the diameter of a group is calculated using distances in G. The constraint on these n groups is that they must be incremental: M_1 is a subset of M_2 ... is a subset of $M_n = V$. The cost of a sequence is the maximum ratio between the diameter of each group M_i and the diameter of a group N_i with i vertices and minimum diameter. This quantity captures the impact of the incremental constraint on the diameters of the groups in a sequence. We give general bounds on the value of this ratio and we prove that the problem of constructing an optimal incremental sequence cannot be solved approximately in polynomial time with an approximation ratio less than 2 unless P = NP. Surprisingly, the related eccentricity problem is in P. We develop an optimal eccentricity algorithm and use it as the basis of a 4-approximation algorithm for the diameter problem. We show that the analysis of our algorithm is tight.

Joint work with Ralf Klasing, Christian Laforest, and Nicolas Thibault

A new proof for Knuth's old sum
Gara Pruesse, Malaspina University-College

Remember the awe you experienced when you saw your first combinatorial proof? Relive the magic! The Reed-Dawson Identity (also known as Knuth's old sum), which states

$$\sum_{k\geq 0}\binom{n}{k}\binom{2k}{k}(-2)^{n-k}=\begin{cases}0 & \text{if } n \text{ odd}\\ \binom{n}{n/2} & \text{if } n \text{ even.}\end{cases}$$

has ten known proofs, which we will review just enough to make them look difficult. Then we'll present a new proof – a combinatorial proof – that not only looks easy, it *is* easy!

Famous and lesser known problems in "elementary" combinatorial geometry. Moshe Rosenfeld, University of Washington

Which problems attain great notoriety and which are delegated to collect dust on a shelf? Elementary problems tend to attract attention because they are very easy to understand and look "solvable". It is a mystery to me why some attract a lot of attention while others lie hibernating waiting for some new fresh ideas.

In their recent interesting book Research Problems in Discrete Geometry (Springer, New York 2005) P. Brass, W. Moser, J. Pach wrote: *"Although Discrete Geometry has a rich history extending more than 150 years, it abounds in open problems that even a high-school student can understand and appreciate. Some of these problems are notoriously difficult and are intimately related to deep questions in other fields of mathematics. But many problems, even old ones, can be solved by a clever undergraduate or a high-school student equipped with an ingenious idea and the kinds of skills used in a mathematical olympiad."*

In this talk I'll survey some "elementary" open problems in discrete geometry. Some of these problems are characterized by constructions yielding lower bounds and upper bounds. For instance, Nelson's problem, coloring the points of R^2 so that points at unit distance have distinct colors is a typical, probably the most popular among these problems. While for this problem, also known as the unit-distance graph problem, the bounds $4 \leq \chi(R^2) \leq 7$

where known over 50 years ago, so far no one was able to improve them. This led to many variations and hundreds of related problems and papers.

In this talk I'll survey some similar problems in combinatorial geometry. Among them:

1. Hadwiger-deBrunner (p, q) problem.

2. Evolution of a problem: Given an angle α. What is the maximum number of lines through the origin in R^d such that among any 3 distinct lines there is at least one pair with angle α between them? When $\alpha = 90$ we proved that it is $2d$. This problem originated from an old problem of P.Erdos and M. Rosenfeld on embedding triangle-free graphs on unit spheres. Its final solution had 3 different versions. One used results of Konyagin a second solution by Vojta Rodl was constructive and the last recent solution by Thomasse used probabilistic methods.

3. The odd-distance graph.

4. Partitioning a square into rectangles.

Hamming distance from an irreducible polynomial
Frank Ruskey, University of Victoria

We initiate an empirical study of the distance of polynomials from being irreducible and from being primitive. This study is based on the tables produced by our recent fast implementation of programs to exhaustively list irreducible and primitive polynomials over the finite fields GF(2), GF(3), GF(4), GF(5), GF(7), and GF(8). In this talk we consider the Hamming distance from a degree $n + 1$ polynomial (with non-zero constant term) to an irreducible polynomial of degree $n + 1$, with all polynomials over GF(2). Up to degree 32, there is no polynomial of distance greater than 3; is there ever one? In an attempt to get a handle on the problem we ask: what is the expected number of length n bitstrings at Hamming distance one from a randomly chosen collection S of odd density bistrings? What if the set S is constrained to invariant under reversal of the bitstrings? We present exact and asymptotic answers to the later two questions and their relation to the initial question about irreducible polynomials.

(In collaboration with Gilbert Lee and Aaron Williams.)

Locating and avoiding errors in software testing
Brett Stevens, Carleton University

The past 15 years have seen a great deal of research on the combinatorial object used in reliability testing: the covering array. In the past few years the model has been adapted to incorporate application relevant issues: known non-interactions, mixed alphabet sizes and mixed strength. We will discuss two very recent adaptations which are surprisingly related: avoiding pairs of known, bad interactions and locating the exact loaction of an error. These are NP-complete for arbitrary situations and polynomial for binary alphabets. In the tractable cases we have characterizations of feasibility and good algorithms.

Symmetries of Venn Diagrams Embedded on the Sphere
Mark Weston, University of Victoria

There are several different chain decompositions of the boolean lattice known. Knuth's favourite is the "Christmas tree pattern", discovered by de Bruijn, van Ebbenhorst Tengbergen and Kruyswijk, which is a simple way of decomposing the lattice into $\binom{n}{n/2}$ rows and $n+1$ columns. Recently, Griggs, Killian and Savage showed how to use this decomposition to create Venn diagrams by turning the decomposition into the dual graph of a Venn diagram. Starting from this work, we show how to modify their chain decomposition to create Venn diagrams that have certain nice symmetries. We show how to create decompositions that map onto themselves by different operations; each such decomposition gives rise to a Venn diagram that, on the sphere, maps onto itself via a symmetry operation. This leads into a discussion of the different types of symmetries possible for Venn diagrams on the sphere. Diagrams can by symmetric under "total" symmetry, where each curve maps onto itself under some operation, or "curve-preserving" symmetry, where each curve maps onto another curve. Time permitting, we'll show various examples of diagrams with different symmetries, and explore some necessary and sufficient conditions for various symmetry groups to be realizable by Venn diagrams.

This is joint work with Brett Stevens (Carleton) and Frank Ruskey (UVic).

Postnikov's Identity and its Generalizations
Laura L. M. Yang, University of Alberta

Postnikov (2004) derived an identity in which a sum over incomplete binary trees was expressed in terms of the number of labelled forests and he asked for a combinatorial proof. Seo (2005) gave a bijective proof. He also generalized Postnikovs identity by finding a closed expression for the generating function for the number of proper vertices in certain families of labelled rooted trees and forests, where a vertex is proper if its label is smaller than the label of any of its descendants. We describe an approach to these results that involves a straightforward inductive argument using a version of the Lambert-Rothe identities.

Generating multi-set permutations in a cool way
Aaron Williams, University of Victoria

In this talk we provide a surprisingly simple and efficient way to generate every permutation of a multi-set. The algorithm runs in constant-amortized time, and has a number of interesting properties. In particular, when specialized to multi-sets containing only two distinct elements, the permutations are generated in the cool-lex ordering due to Ruskey-Williams, which appears in Don Knuth's new volume of The Art of Computer Programming.