A Rigorous Time-Domain Analysis of Full–Wave Electromagnetic Cloaking (Invisibility) *†

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Abstract

There is currently a great deal of interest in the theoretical and practical possibility of cloaking objects from the observation by electromagnetic waves. The basic idea of these invisibility devices [2, 3, 4], [5] is to use anisotropic *transformation media* whose permittivity and permeability $\varepsilon^{\lambda\nu}, \mu^{\lambda\nu}$, are obtained from the ones, $\varepsilon_0^{\lambda\nu}, \mu_0^{\lambda\nu}$, of isotropic media, by singular transformations of coordinates. In this paper we study electromagnetic cloaking in the time-domain using the formalism of time-dependent scattering theory [6]. This formalism provides us with a rigorous method to analyze the propagation of electromagnetic wave packets with finite energy in *transformation media*. In particular, it allows us to settle in an unambiguous way the mathematical problems posed by the singularities of the inverse of the permittivity and the permeability of the *transformation media* on the boundary of the cloaked objects. Von Neumann's theory of self-adjoint extensions of symmetric operators plays an important role on this issue. We write Maxwell's equations in Schrödinger form with the electromagnetic propagator playing the role of the Hamiltonian. We prove that every self-adjoint extension of the electromagnetic propagator in a *transformation medium* is the direct sum of a fixed self-adjoint extension in

*PACS classification scheme 2006: 41.20.Jb, 02.30.Tb,02.30.Zz, 02.60.Lj):
†Research partially supported by CONACYT under Project P42553F.
‡Fellow Sistema Nacional de Investigadores.

the exterior of the cloaked objects, that is unitarily equivalent to the electromagnetic propagator in the homogeneous medium, with some self-adjoint extension of the electromagnetic propagator in the interior of the cloaked objects. Each of these self-adjoint extensions corresponds to a possible unitary time evolution for finite energy electromagnetic waves. As is well known, the fact that time evolution is unitary assures us that energy is conserved. It is also well known that choosing a particular self-adjoint extension of the electromagnetic propagator of the cloaked objects amounts to choosing some boundary condition on the inside of the boundary of the cloaked objects. In other words, any possible unitary dynamics implies the existence of some boundary condition on the inside of the boundary of the cloaked objects. The particular boundary condition that has to be taken will depend on the specific properties of the metamaterials used to build the transformation media as well us on the properties of the media inside the cloaked objects. Our results mean that the electromagnetic waves inside the cloaked objects are not allowed to leave them, and viceversa, electromagnetic waves outside can not go inside. Furthermore, we prove that the scattering operator is the identity. This implies that for any incoming finite-energy electromagnetic wave packet the outgoing wave packet is precisely the same. In other words, it is not possible to detect the cloaked objects in any scattering experiment where a finite energy wave packet is sent towards the cloaked objects, since the outgoing wave packet that is measured after interaction is the same as the incoming one. Our results give a rigorous proof of cloaking of passive and active devices from observation with electromagnetic waves. A rigorous prof of cloaking has already been given by [1] where fixed frequency waves were studied, i.e., in the frequency domain, and cloaking, at any frequency, with respect to the measurement of the Cauchy data on a surface that encloses the cloaked object was proven. Our results here give an alternative treatment of this problem in the time-domain.

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