The Inverse Time-dependent Coefficient Identification Problem in Bio-Heat Transient Flow Equation

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Abstract

The governing equation for the heat transfer process within the human body tissue, namely

$$\Delta T - P_f T + S = \frac{\partial T}{\partial t}\,, \tag{1}$$

has important applications in many biomedical investigations. Among other theoretical aspects that we are concerned with regard to this equation, the perfusion coefficient P_f receives a particularly important interest because of its physical meaning, that is $P_f = \frac{w_b c_b \mathcal{L}^2}{k_t}$, where w_b is the perfusion coefficient of blood, c_b is the heat capacity of blood, $\mathcal{L}^2$ is the characteristic dimension of the tissue and k_t is the thermal conductivity of the tissue.

In all of our analysis we consider the non-steady state time-dependent case with P_f dependent on time, as well as S dependent both on space and in time.

Within this context, the following inverse problem focuses our interest:

Find the temperature $T(x,t)$, as a function in $\mathscr{C}^{2,1}(\Omega\times(0,t_f])\bigcap\mathscr{C}^{1,0}(\overline{\Omega}\times[0,t_f])$, and the perfusion time-dependent coefficient $P_f(t) > 0$, $\forall t > 0$, satisfying both one and two dimensional bio-heat equation

$$\Delta T - P_f(t) T = \frac{\partial T}{\partial t} \tag{2}$$

where, besides the standard smooth enough initial conditions and Dirichlet boundary conditions, we consider the mass measurement

$$\int_\Omega T(x,t)dx = E(t)\,, \quad \forall t \in (0,t_f]\,. \tag{3}$$

At the conference the robustness of the solution of this inverse problem will be presented both from analytical and numerical stand point.

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