A Numerical Study Of Some Globally Convergent Algorithms For Solving Non-Overdetermined Coefficient Inverse Problems

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Abstract

Some globally convergent algorithms for the numerical solution of non-overdetermined coefficient inverse problems are studied. Two methods are used to construct such algorithms. In the convexification method developed by the author together with M. V. Klibanov, the Carleman weight functions associated with Carleman estimates of the second-order differential operator are utilized to provide the global convergence of numerical solutions to the solution of an original inverse problem. The resulting convexification algorithms exploit either the methods of nonlinear constrained optimization or successive approximations. The results of numerical experiments with the inverse problems of magnetotelluric sounding and microwave imaging are demonstrated. It is concluded that although the convexification algorithms allow for the stable reconstruction of coefficients, they do not provide the high spatial and contrast resolution, especially in high dimensions. The main reason is that the convexification method is based on a sufficiently rough approximation of the integro-differential operator in an auxiliary Cauchy problem by truncating an improper integral and projecting the solution of this problem onto a finite-dimensional

space. As a result, both the accuracy and convergence rates are affected, sometimes severely. Such deficiencies motivate further studies. In an on-going research project pursued in collaboration with A. Nachman and A. Tamasan, another approach is developed. The key idea is to transform an original inverse problem to an initial value problem for a parametric family of the first-order ordinary differential equations. Unlike the convexification method, in this approach, the exact operator generated by the initial value problem is used when constructing globally convergent successive approximations utilizing Picards method. To provide a more accurate numerical treatment of improper integrals, the initial value problem is formulated for the increment function with respect to a given background medium, and it is solved together with the forward when computing a successive approximation. The first results of numerical experiments with a 1D model inverse problem of acoustic imaging are reported to demonstrate the computational feasibility and effectiveness of this approach.