Perturbation formulas for phase velocity and polarization of Rayleigh waves in prestressed anisotropic media

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Rayleigh waves are elastic surface waves which propagate along the traction-free surface with the phase velocity in the subsonic range, and whose amplitude decays exponentially with depth below that surface. Such waves serve as a useful tool in nondestructive characterization of materials. The problem there is what material information we could obtain if we could measure accurately Rayleigh waves propagating in any direction on the traction-free surface.

Herein we consider Rayleigh waves propagating along the traction-free surface of a macroscopically homogeneous, anisotropic, prestressed half-space. Assuming that the deviation of the prestressed anisotropic medium from a comparative 'unperturbed', unstressed and isotropic state be small, we investigate the perturbation of the phase velocity and of the polarization of Rayleigh waves caused by the anisotropic part $\mathbb{A} = (a_{ijkl}), (i, j, k, l = 1, 2, 3)$ of the incremental elasticity tensor and by the initial stress $\overset{\circ}{\boldsymbol{T}} = \big(\overset{\circ}{T}_{ij}\big), (i, j = 1, 2, 3)$. Here, we do not put any restriction on the symmetries of $\mathbb{A}$ and $\overset{\circ}{\boldsymbol{T}}$ except for the physically natural conditions $a_{ijkl} = a_{klij} = a_{jikl}$ and $\overset{\circ}{T}_{ij} = \overset{\circ}{T}_{ji}$, so that $\mathbb{A}$ has 21 independent components.Two physical quantities are determined by the polarization of Rayleigh waves. One is the polarization ratio, which is the ratio of the maximum longitudinal component to the maximum normal component of the displacements of the Rayleigh waves at the surface, and the other is the phase shift, which is the shift in phase between the longitudinal component and the normal component of the displacements.

In this talk we present perturbation formulas for the phase velocity, the polarization ratio and the phase shift, which are correct to first order in the components of $\mathbb{A}$ and $\overset{\circ}{\boldsymbol{T}}$. For definiteness, we choose a Cartesian coordinate system such that the material half-space occupies the region $x_3 \leq 0$, whereas the 1- and 2-axis are arbitrarily chosen. (Then $\overset{\circ}{T}_{i3} = 0$ for $i = 1, 2, 3$, and hence $\overset{\circ}{\boldsymbol{T}}$ has 3 independent components.) The following consequences immediately follow from the perturbation formulas:

1. Only four components a_{2222}, a_{2233}, a_{3333} and a_{2323} of $\mathbb{A}$ and one component $\overset{\circ}{T}_{22}$ of $\overset{\circ}{\boldsymbol{T}}$ can affect the first-order perturbation of the phase velocity of Rayleigh waves propagating in the direction of the 2-axis on the surface $x_3 = 0$.
2. Only the same components of $\mathbb{A}$ as the above but no component of $\overset{\circ}{\boldsymbol{T}}$ can affect the first-order perturbation of the polarization ratio of such Rayleigh waves.
3. Only two components a_{2223} and a_{3323} of $\mathbb{A}$ and no component of $\overset{\circ}{\boldsymbol{T}}$ can affect the first-order perturbation of the phase shift of such Rayleigh waves.

We discuss the problem of determining $\mathbb{A}$ and $\overset{\circ}{\boldsymbol{T}}$ by making measurements of perturbation of Rayleigh waves which propagate in any direction on the traction-free surface. We show that, to first order of $\mathbb{A}$ and $\overset{\circ}{\boldsymbol{T}}$, the totality of phase velocities of Rayleigh waves propagating in all the directions on the traction-free surface carries information only on 13 components of $\mathbb{A}$ and on all the 3 components of $\overset{\circ}{\boldsymbol{T}}$. On the other hand, it can be proved that the totality of the phase shifts of those Rayleigh waves carries information only on the remaining 8 components of $\mathbb{A}$.