On the recovery of potential field anomalies on the surface of a sphere from incomplete and noisy data taken at an altitude above their source

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Abstract

The estimation of potential fields such as the gravitational or magnetic potential at the surface of a spherical planet from noisy observations taken at an altitude over an incomplete portion of the globe is a classic example of an ill-posed inverse problem. We show that this potential-field estimation problem has deep-seated connections to Slepian's spatiospectral localization problem which seeks bandlimited spherical functions whose energy is optimally concentrated in some closed portion of the unit sphere. This allows us to formulate an alternative solution to the traditional damped least-squares spherical harmonic approach in geodesy, whereby the source field is now expanded in a truncated Slepian function basis set. We discuss the relative performance of both methods with regard to standard statistical measures such as bias, variance and mean squared error, and pay special attention to the algorithmic efficiency of computing the Slepian functions on the region complementary to the axisymmetric polar gap characteristic of satellite surveys. The ease, speed, and accuracy of our method make the use of spherical Slepian functions in earth and planetary geodesy practical.