Inverse Spectral Problems in Rectangular Domains

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Abstract. This is a report on joint work with Gregory Eskin. We consider the Schrödinger operator $-\Delta + q$ in domains of the form $D = \{x \in R^n : \; 0 \leq x_i \leq a_i, \; i = 1, .., n\}$ with either Dirichlet or Neumann boundary conditions on the faces of D, and study the constraints on q imposed by fixing the spectrum of $-\Delta + q$ with these boundary conditions. We work in the space of potentials, q, which become real-analytic on R^n when they are extended evenly across the coordinate planes and then periodically. Our results have the corollary that there are no continuous isospectral deformations for these operators within that class of potentials when the a_jś are rationally independent.

This work is based on new formulas for the trace of the wave group in this setting. It is an extension of the proof of similar results for periodic boundary conditions (Eskin-Ralston-Trubowitz, CPAM **37**). The wave trace with Dirichlet or Neumann boundary conditions is significantly different from the trace for periodic boundary conditions, but the new terms in the trace "telescope" in a way which simplifies their contribution to the singularities of the trace. The resulting formulas reveal close relations between these traces and the periodic trace, and they are also make it possible to compute the singularities in the trace in a way that identifies contributions with the underlying geometry. In addition to the inverse spectral results these formulas lead to asymptotic expansions for the traces of the wave and heat kernels on rectangular domains.