Uniqueness for discontinuous coefficients in an inverse problem for the heat equation

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ABSTRACT

Let $\Omega \subset \mathbb{R}^n$ be a bounded connected open set that satisfies a natural geometric condition. Let Ω_0 and Ω_1 be two non-empty open subsets of Ω such that $\Omega_0 \subset\subset \Omega$ and $\Omega_1 = \Omega \setminus \bar{\Omega}_0$. We denote by $S = \bar{\Omega}_0 \cap \bar{\Omega}_1$ the interface. We consider the heat equation with a discontinuous diffusion coefficient c at the interface S:

$$\begin{aligned} \partial_t y - \nabla \cdot (c \nabla y) &= 0 && \text{in } (0,T) \times \Omega \\ y(t,x) &= h(t,x) && \text{in } (0,T) \times \delta\Omega \\ y(0,x) &= y_0 && \text{in } \Omega \end{aligned}$$

We assume that the diffusion coefficient c is smooth in each domain Ω_j and that we can measure both the normal flux $\partial_n \partial_t y$ on $\gamma \subset \partial\Omega$ on the time interval (t_0, T) and y in Ω at time $T' \in (t_0, T)$.

But we do not know the interface S which is an unknown in this inverse problem.

We shall prove uniqueness for S and the diffusion coefficient c with only one given boundary condition.

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