Solution of Ill-Posed Problems via Adaptive Grid Regularization

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Abstract:

When studying (linear or nonlinear) ill-posed problems

$$F(x) = y\,, \qquad F : \mathcal{D}(F)(\subset \mathcal{X}) \to \mathcal{Y}\,,$$

where usually only noisy measurements y^δ of y with $\|y^\delta - y\| \le \delta$ are given, $\mathcal{Y}$ is a Hilbert space and $\mathcal{X}$ is a Banach space, it is well known by now that standard regularization methods are not appropriate for ill-posed problems with discontinuous solutions, since they have a smoothing effect on regularized solutions.

If one expects discontinuous solutions, special care has to be taken in choosing the regularization method. Bounded variation regularization has turned out to be an effective method when dealing with such problems. An other approach is regularization for graph and surface representations. Based on these ideas, the author recently developed a new method, namely *adaptive grid regularization*, an iterative method, where local grid refinement techniques are combined with adaption of the regularizing norm after each iteration.

The method has already been successfully applied to linear integral equations in 1D and 2D as well as to 1D parameter estimation problems. The numerical results show that this method is an efficient and fast tool to identify discontinuities of solutions of ill-posed problems.

In this talk, we present a convergence analysis for the adaptive grid regularization method combined with Tikhonov regularization and new numerical results for the problem of identifying a temperature dependent heat conductivity from a single boundary measurement:

$$\begin{aligned} -\operatorname{div}(a(u)\nabla u) &= f \quad \text{in } \Omega \\ a(u)\frac{\partial u}{\partial n} &= h \quad \text{on } \Gamma = \partial\Omega \end{aligned}$$

Here Ω is an open bounded convex subset of $\mathbb{R}^d$ $(d = 1, 2, 3)$ with Lipschitz boundary Γ, $f \in L^2(\Omega)$, $h \in L^2(\Gamma)$, and the parameter a satisfies the conditions

$$\begin{aligned} &0 < \underline{a} < a < \overline{a} < \infty \quad \text{and} \\ &a \text{ is continuous in all but at most countably many points}\,. \end{aligned}$$