Jump Estimation in Inverse Problems - Distributional Results

Axel Munk, Institute for Mathematical Stochastics, *Georg-August Universität Göttingen*

April 13, 2007

We adress the problem of estimating a jump function in the context of an inverse regression problem $Y_i = Kf(x_i) + \epsilon_i$, $i = 1, \cdots, n$, where K is a known (linear) integral operator and $f : [0,1] \longrightarrow \mathbb{R}$ is the unknown function to be estimated. The x_i are (regular, possibly random) design points. It turns out that here a $\sqrt{n}$-rate of convergence is generic and minimax, provided the kernel of K is bounded and continuous. In fact, the jump locations together with the jump sizes are asymptotically multivariate normal. To this end we require an identifiabilty condition related to the theory of radial basis functions on the kernel K, which turns out to be crucial for recovering jump functions in nosiy inverse problems.

Asymptotic normality can be used to construct confidence bands for jump functions or for a piecewise linear regression function in multiphase regression.

We stress that our analysis for jump spaces is completely different to the situation where the underlying function space is of some smoothing type, such as a Sobolev space, where the spectral behaviour of K determines the asymptotics [**?**]. We show that for jump spaces the localisation behaviour of the kernel determines the rate of convergence, rather than the spectral behaviour. In this sense a bounded integral kernel is most difficult. We obtain, e.g. for singular kernels with decay of the singularity of the order $|x|^{-\alpha}$, $\alpha \in [1/2, 1)$ the minimax rate (which is attained by the *Jplse*) as $n^{-1/\min(2,(3-2\alpha))}$.

Motivated by a problem from material science, we extend this to estimation problems with certain nonlinear operators. More specifically we show similar results for the class of generalized Hammerstein equations of the type

$$K_\varphi(f)(\cdot) \; = \; \int K(x, \cdot)\varphi \circ f(x)dx,$$

for some φ injective, C^1. It is an open and challenging problem how general nonlinear problems can be treated.

References

[1] L. Boysen, V. Liebscher, A. Munk, O. Wittich, *Scale space consistency of piecewise constant least squares estimators - another look at the regressogram*, IMS Lecture Notes (2006), to appear.

[2] L. Boysen, L., A. Kempe, V. Liebscher, A. Munk, O. Wittich, *Consistencies and rates of convergence of jump-penalized least squares estimators*, Ann. Statist, in revision.

[3] A. Goldenshluger, A.Tsybakov, A. Zeevi, *Optimal change-point estimation from indirect observations*, Ann. Statist. **34** (2006), 350-372.