Dimensionality reduction and polynomial chaos acceleration of Bayesian inverse problems

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The Bayesian approach to inverse problems provides a foundation for inference from noisy data and uncertain forward models, a natural mechanism for regularization in the form of prior information, and a quantitative assessment of uncertainty in the inverse solution. With computationally intensive forward models such as PDEs, however, the cost of repeated likelihood evaluations may render a Bayesian approach prohibitive. This problem is compounded by high dimensionality, as when the unknown is a spatiotemporal field. We address these difficulties in the case of Gaussian process priors: first, by using a Karhunen-Loève (KL) expansion to express the inverse solution as a joint distribution over the KL mode strengths; then by using a Galerkin/polynomial chaos construction to efficiently propagate prior uncertainty through the forward model. The result is a lower-dimensional surrogate posterior which may be explored at negligible cost. We demonstrate this approach on transient diffusion equations with unknown inhomogeneous diffusivities.