Nonlinear integral equations for an inverse electromagnetic scattering problem

Martin Pieper
Institute for Numerical and Applied Mathematics
Georg-August-University Goettingen

March 2007

Abstract

In 2005, Kress and Rundell [1] developed a new method to solve an inverse boundary value problem for Laplace equation. They showed, that the inverse problem is equivalent to a system of nonlinear integral equations, which is solved iteratively. This idea was carried over to inverse acoustic scattering problems by Ivanyshyn and Kress [2]. With some modifications, the main parts of their work can be used to show, that the inverse electromagnetic scattering problem is also equivalent to a system of nonlinear integral equations. Avoiding the usage of the hypersingular electric dipole operator leads to a system of two equations. The first one is the well posed magnetic field equation, whereas the second one is severely ill posed.

In the talk, after presenting the main parts of the equivalence proof, we show in detail how to solve iteratively the corresponding system of integral equations: With an initial guess to the boundary, we first solve the magnetic field equation for the unknown density. For this, we have developed a spectral method, which is based on Wienert's and Graham and Sloan's work on acoustic scattering ([3] and [4]). After that we solve a linearised version of the second equation to get a new approximation to the unknown scatterer and iterate this procedure.

References

[1] R. Kress and W. Rundell *"Nonlinear integral equations and the iterative solution for an inverse boundary value problem"*, Inverse Problems 21, 1207-1223 (2005)

[2] O. Ivanyshyn and R. Kress *"Nonlinear integral equations in inverse obstacle scattering"*, n: Mathematical Methods in Scattering Theory and Biome-

dical Engineering (Fotiatis, Massalas, eds). World Scientific, Singapore, 39-50 (2006)

[3] Wienert L., *Die numerische Approximation von Randintegraloperatoren fuer die Helmholtzgleichung im* $\mathbb{R}^3$, PhD Thesis, Goettingen 1990

[4] Graham I.G. and Sloan I.H., *Fully discrete spectral boundary integral methods for Helmholtz problems on smooth closed surfaces in* $\mathbb{R}^3$, Numer. Math. (2002) 92, S. 289-323