Invisibility and Inverse problems

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Abstract

There has recently been considerable interest in the possibility, both theoretical and practical, of invisibility of objects to different types of waves. We construct several examples of cloaking enclosures covered with anisotropic materials. These examples have a close connection to earlier works carried out for the case of static Maxwell's equations (at zero frequency) [1,2], a case that is very important in electrical impedance tomography (EIT). These results have also a close connection to counter examples for inverse problem. For instance, consider the Calderon problem, that is, whether the Dirichlet-to-Neumann map determines uniquely the conductivity. The problem has a positive answer in all dimensions $n \geq 2$ if the conductivity is isotropic (under suitable regularity hypothesis) and in 2D, it also known for anisotropic conductivities. However, in all of these results it is assumed that the conductivity is bounded both below and above by strictly positive constants. If this condition is violated, one can cover any object with a properly chosen anisotropic material so that the covered object appears in all boundary measurement similar to a domain with constant conductivity. Clearly, this kind of counter example gives us theoretical instructions how to cover an object so that it appears 'invisible" in zero frequency measurements. In this talk we consider similar kind of result for all frequencies.

We review the results on the counter examples based on singular transformations that push isotropic electromagnetic parameters forward into singular, anisotropic ones. We define the notion of finite

energy solutions of Maxwell's equations for such singular electromagnetic parameters, and study the behavior of the solutions on the entire domain, including the cloaked region and its boundary. Analyzing the behavior of such solutions inside the cloaked region, we show that, depending on the chosen construction, there appear new "hidden" boundary conditions at the surface separating the cloaked and uncloaked regions. We also consider the effect on invisibility of active devices inside the cloaked region, interpreted as collections of sources and sinks or internal currents. When these boundary conditions are overdetermined, we see that Maxwell's equation do not have finite energy solutions for the generic internal currents. We will consider in detail the existence of the finite energy solutions when cloaking a ball or an infinite cylinder. When cloaking an infinite cylinder, we show that the finite energy solutions solutions exist if the coating material is lined from inside with a surface satisfying the soft and hard (SHS) boundary condition. Surprisingly, without such a lining finite energy solutions may not exists even if there are no internal sources.

The results have been done in collaboration with Allan Greenleaf (Univ. of Rochester), Yaroslav Kurylev (Univ. of Loughborough, UK), and Gunther Uhlmann (Univ. of Washington).

References:

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