Identification and Characterization of a Moving Source in a General Parabolic Differential Equation

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Abstract

We discuss an inverse source problem for a general parabolic differential equation in $\mathbb{R}^n \times \mathbb{R}$ with variable coefficients and a source whose strength and support may vary with time. We demonstrate that a knowledge of the solution on any bounded open set M in $\mathbb{R}^n$ located away from the source for any fixed time t determines the so-called "carrier support" [as originally defined and presented in, Notions of support for far fields, John Sylvester, Inverse Problems, 2006] (a nontrivial subset of the support of the true source) at that coincident time. Additionally, we provide a reconstruction algorithm which can locate the time-varying position of the carrier support of the assumed unknown source with extremely few discrete (possibly nonuniform) measurements taken on such an open set over a wide range of regulartity classes of the source.

Lastly, we provide a few numerical examples which illustrate the efficacy and robustness of this location and tracking method.

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