Some Inverse Problems for Partial Differential Equations

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Abstract

We will consider the following topics:

1. An inverse problem of the recovery of the initial condition in the parabolic equation $u_t = L(x,t,D)u + f(x,t)$ from the Dirichlet and Neumann boundary data. Here $L(x,t,D)$ is a general elliptic operator of the second order. Logarithmic stability estimate will be presented for the case when $x \in \Omega$, where Ω is a bounded domain [1]. This is an extension of the previously known result for case of the self adjoint operator $L(x,D)$ with t-independent coefficients, which was obtained by the logarithmic convexity method. While the first result is for the finite domain, the second stability estimate is for an infinite domain. In this case a new Carleman estimate is derived.

2. A coefficient inverse problem for the non-stationary transport equation. We prove uniqueness result for the case when the absorption coefficient is recovered from boundary measurements at the time interval $(0,T)$. The main difficulty here is that one needs to properly extend the solution of the transport equation in $(-T,T)$ in such a way that boundary integrals over $\{t=0\}$ in the Carleman estimate would cancel out.

3. Numerical results for the inverse problem of the recovery of the initial condition of a hyperbolic equation from the Dirichlet and Neumann boundary data [2]. We present an analog of the quasi-reversibility method of R. Lattes and J.-L. Lions, prove its convergence and demonstrate numerical results.

4. A globally convergent convexification algorithm for some coefficient inverse problems. The algorithm will be presented along with numerical results [3].

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References

1. M.V. Klibanov, Estimates of initial conditions of parabolic equations and inequalities via lateral Cauchy data, QTRemInverse Problems, 22, 2006, 495-5 14.

2. C. Clason and M.V. Klibanov, The quasi-reversibility method for thermoacoustic tomography in a heterogeneous medium, submitted for publication.

3. M.V. Klibanov and A. Timonov, Numerical studies on the globally convergent convexification algorithm in 2D, QTRemInverse Problems, 23, 2007, 123-138.