A Mumford-Shah level-set approach for the inversion and segmentation of SPECT data

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In this talk we present a level-set approach for the reconstruction of an activity function f and a density function μ from SPECT data (Single Photon Emission Computerized Tomography) modeled by the attenuated Radon transform

$$y(s,\omega) = A(f,\mu) = \int_{t\in\mathbb{R}} f(s\omega + t\omega^{\perp}) \exp\Big(- \int_{\tau=t}^{\infty} \mu(s\omega + \tau\omega^{\perp}) \mathrm{d}\tau \Big) \mathrm{d}t \ .$$

For the medical application at hand, we consider it reasonable to restrict the reconstructions to activity functions f and density functions μ which are constant with respect to a partition of the body. Hence, we determine a pair of piecewise constant functions (f,μ) from possibly noisy SPECT data. Simultaneously, a segmentation of the reconstructed density and activity is obtained. The segmenting contours and the corresponding density and activity functions are found as minimizers of a Mumford-Shah like functional over the set of admissible contours and – for a fixed pair of contours – over the space of piecewise constant density and activity functions which may be discontinuous across the contour. Shape sensitivity analysis is used to find a descent direction for the cost functional which leads to an update formula for the contour in the level-set framework.