Frechet-differentiation of functions of operators with application to testing the equality of covariance operators

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Abstract

It is well-known that the sample covariance operator converges in distribution in the Hilbert space of Hilbert-Schmidt operators, and that this result entails the asymptotic distribution of simple eigenvalues and corresponding eigenvectors.

Several estimators and test statistics for the analysis of functional data require the asymptotic distribution of eigenvalues and eigenvectors of certain functions of the sampl covariance operator. To obtain such a result, apparently the asymptotic distribution of such a function of the sample covariance operator is a prerequisite. In this paper we discuss the Frchet-derivative of functions of operators and an ensuing delta-method to solve this problem.

The results are applied to obtain the asymptotic distribution of a test statistic for testing the equality of two covariance operators.