Optimization method for solving two dimensional inverse problem for hyperbolic equation

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Abstract

Consider two dimensional inverse problem for hyperbolic equation $u_{tt} = \Delta_{x,y}u - q(x,y)u$.

According projection method [1] consider the system of integral equations

$$\mathbf{v}(x,t) = \frac{1}{2}\Big[\mathbf{f}(t+x)+\mathbf{f}(t-x)\Big]+\frac{1}{2}\int\limits_0^x \int\limits_{t-x+\xi}^{t+x-\xi} (\mathcal{N}+\widehat{\mathcal{Q}})\mathbf{v}(\xi,\tau)\,d\tau\,d\xi, \qquad (1)$$

$$\mathbf{q}(x) = \mathbf{f}(x) + \int\limits_0^x \int\limits_{-x+\xi}^{x-\xi} (\mathcal{N}+\widehat{\mathcal{Q}})\mathbf{v}(\xi,\tau)\,d\tau\,d\xi, \quad x \in (0,T). \qquad (2)$$

Where $\mathbf{v}$, $\mathbf{f}$, $\mathbf{q}$ – vectors $(2M+1)$, $\mathcal{N}$, $\mathcal{Q}$ – matrices $(2M+1)\times(2M+1)$.

Direct problem. *Find* $\mathbf{v} \in L_2(\Delta(T))$ *by* $\mathbf{f} \in L_2(T)$, $\mathbf{q} \in L_2(T)$ *satisfying (1).*

Inverse problem. *Find* $\mathbf{q} \in L_2(T)$ *by* $\mathbf{f} \in L_2(T)$ *satisfying (2). (In (2) function* $\mathbf{v}(x,t) = \mathbf{v}(x,t;\mathbf{q})$ *is determined by* $\mathbf{q}(x)$ *from (1)).*

We suggest that $\mathbf{q} \in L_2(T)$, if $q_{(i)}(x) \in L_2(0,T)$ for all $|i| \le M$ and $\mathbf{v} \in L_2(\Delta(T))$ if $v_{(i)}(x,t) \in L_2(\Delta_*(T,0))$, $|i| \le M$.

We define norms

$$\|\mathbf{q}\|^2_{L_2(T)} = \int\limits_0^T \Big(\sum_{|i|\le M} q^2_{(i)}(x)\Big)\,dx,$$

$$\|\mathbf{v}\|^2_{L_2(\Delta(T))} = \int\limits_{\Delta_*(T,0)} \Big(\sum_{|i|\le M} v^2_{(i)}(x,t)\Big)\,dt\,dx$$

Let $\|\mathbf{q}\|_{L_2(T)} \leq K_2, \qquad \|\mathbf{f}\|_{L_2(T)} \leq Q_2$ are determined. We denote

$$\Omega_2(M,T,K_2) =$$
$$= \big\{ \mathbf{q} \mid q_{(j)}(x) \in L_2[0,T], \;\; j = \overline{-M,M}, \;\; \|\mathbf{q}\|_{L_2(T)} \leq K_2 \big\}.$$

Let $\mathbf{p}(x) \in \Omega_2(M,T,K_2)$ and

$$\boldsymbol{\eta}(x) = \mathbf{p}(x) - \int\limits_0^x \int\limits_{-x+\xi}^{x-\xi} (\mathcal{N} + \widehat{\mathcal{P}})\mathbf{v}(\xi,\tau)\, d\tau\, d\xi - \mathbf{f}(x), \quad x \in (0,T).$$

Consider functional of new type

$$J[\mathbf{p}] = \int\limits_0^T \sum_{|n| \leq M} [\eta_{(n)}]^2(x)\, dx.$$

We obtained a strong convergence rate of the steepest descent method

$$\|\mathbf{p}^{(n+1)} - \mathbf{q}\|^2_{L_2(T)} \leq C_6 J[\mathbf{p}^{(0)}] \exp\Big\{ -\frac{n}{4C_4C_5} \Big\}.$$

Where $C_4,\ C_5,\ C_6$ are constants of $M,\ T,\ \|\mathbf{q}\|^2_{L_2(T)},\ \|\mathbf{f}\|_{L_2(T)}$.

References

[1] Kabanikhin S. I. and Iskakov K. T. Justification of the steepest desent method for the integral statement of an inverse problem for a hyperbolic equation // *Siberian Mathematical J.* 2001. V. 42. No 3. P.478–494