Learning dictionaries for sparse representation

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Abstract

The choice of regularisation functional has been a subject of active research for the last few decades, going from the basic energy term $\lambda\|\underline{m}\|_2^2$ of Standard Tikhonov (ST) regularisation, through smoothness term as in Generalised Tikhonov (GT) $\lambda\|L\underline{m}\|_2^2$, and then advanced to more robust measures, such as Total Variation $\lambda\|\underline{m}\|_1$ in ℓ_1 norm, Wavelet Sparsity $\lambda\|W\underline{m}\|_p^p$ in ℓ_p norm ($p \leq 1$) or Huber-Markov. The main drawback of these functionals is that some ad-hoc assumptions as for the behaviour of the solution are taken. Solutions derived from such regularisation functional guessing approach are intrinsically subjective to their arbitrary choice and therefore prone to be distorted from the true model pursued. An alternative approach is to learn the regularisation term from given authentic examples. This recent approach employs sparse representation of the regularisation term $\lambda\|\underline{u}\|_0^0$ for $\underline{m} = D\underline{u}$, where D is an overcomplete dictionary of basis functions. The underlaying assumption in this case is that the true solution is describable by a small number of parameters (principle of parsimony). Thus, the ℓ_0 norm, which effectively counts the number of non-zeros entries in $\underline{u}$, would favour solutions in which $\underline{u}$ is sparse. In this framework the regularisation functional is constructed implicitly, liberating the solution from any artificial and undesirable restrictions of any specific Hilbert space associated with the choice of an explicit functional, albeit allowing more effective and efficient restriction of the solution space using trust-worthy considerations which were acquired from true examples. The choice of overcomplete dictionary is unfortunately a

NP hard problem. Several combinatoric approaches have been proposed for this task in the recent years. Here we propose an innovative constraint optimisation-based approach for updating the dictionary of a sparse-represented regularised inverse problem. The application of this approach for image deblurring is presented and compared with results from state of the art regularisation methods which were available so far.