Algorithms for inverse problems with joint sparsity constraints

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Abstract:

Signals and images can often be well-approximated by sparse representations in terms of a suitable basis or frame, i.e., many coefficients of the expansion vanish. A recent direction in inverse problems uses this preknowledge about the signal (image, function) to be recovered and poses sparsity constraints on the regularized solution. This is modeled by a functional that includes a weighted ℓ_1-norm of the frame coefficients. It was shown by Daubechies, Defrise and DeMol that a minimizer of this functional can be computed by a soft-tresholded Landweber iteration.

The above setting can be generalized to vector-valued functions (signals), where the vector components possess sparse representations with a common sparsity pattern, i.e., the non-zero coefficients are at the same location in each component. A typical example are color images, where each color channel has a sparse representation in terms of wavelets, say, and additionally the relevant coefficients are at the same locations due to common edges in all channels. Such joint sparsity assumptions can be modeled in a functional by using weighted ℓ_1 norms of componentwise ℓ_q norms of frame coefficients. In generalization of the approach by Daubechies, Defrise and DeMol, solutions of linear inverse problems with such joint sparsity regularization constraints can again be computed by soft thresholded Landweber algorithms.

Suitable weights appearing in the weighted ℓ_1-norm can be chosen adaptively by constructing a functional both in the frame coefficients and the weights, which has to be minimized. Hereby, the weights can be interpreted as indicators of the sparsity pattern. The resulting functional has some connections to hard and firm thresholding.

Two algorithms for the minimization of this functional are discussed:

- alternating between a minimization with respect to the frame coefficients and the weights;
- damped hard or firm thresholded Landweber iterations.

Both algorithms converge.

Numerical experiments in color image restoration are presented.