Boundary conditions for nodestructive evaluation based on stationary infrared thermography

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Various tomographic methods (based e.g., on X-rays, electrical impedance, ultrasounds, eddy currents, thermal imaging) can be used to investigate damages over the inaccessible surface of a thin conducting plate, in order to prevent critical situations in which, for example, a specimen should not support excessive pressure.

In particular, stationary models in thermal imaging are relevant in both the experimental frameworks of a constant-in-time heat flux $\Phi(x)$ kept on until the system reaches a stationary status, and thermal waves. In the latter case, a periodic heat flux $\Phi(x,t) = \Phi_0(x) + \Phi_1(x)\sin(\frac{2\pi}{\tau}t) + \ldots$ gives rise to a periodic temperature profile $u(x,t)$ having the same period τ, apart of a more or less fast deacaying term. Heat equation can be Fourier transformed into a sequence of Helmoltz-type elliptic equations. The equation of order zero, in particular, is Laplace's equation. Thus, at least in theory, solving an inverse problem for Laplace's equation can provide an answer to the main problem of detecting damages from observed thermal data.

Different damage aftereffects are modeled by endowing the Laplace's equation with suitable conditions placed on the inaccessible boundary. Here, we deal with two distinct kinds of boundary properties in thermal models:

1. The damage does not affect the shape of the specimen, but it leaves effects on its transmissivity properties. This possibility leads to the largely studied problem of recovering coefficients in Robin boundary conditions. Recently, in [2] and [3] we supposed that the damage gives origin to non-linearities in the cooling law. We considered uniqueness, stability, and numerical solution of the problem of recovering these nonlinearities from thermal data.

2. As observed in literature, the effect of cooling is neglectable in many cases. The presence of non-insulation terms in the boundary conditions is in fact quantitatively important only for some materials in particular environment, see [1]. In these cases, we studied the problem of recovering small perturbations in the shape of the specimen from thermal data when other physical parameters in boundary conditions are known [4].

References

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