Inverse problems for the Schrodinger equations with time-dependant electromagnetic potentials and the Aharanov-Bohm effect

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Abstract

We consider nonstationary Schrödinger equation with time-dependent electromagnetic potentials in space-time domain $D \subset R^{n+1}$ containing obstacles. In general, the domain is multi-connected and the obstacles are moving obstacles. The multi-connectness of the domain leads to the Aharonov-Bohm effect, and the presence of moving obstacles leads to the Aharonov-Bohm effect caused by the electric fluxes, as well by the magnetic fluxes.

We introduce gauge invariant boundary data. The main theorem is that the gauge invariant boundary data uniquely determine the electromagnetic potentials up to a gauge transformation. The fact that the gauge invariant boundary data can distinguish between two not gauge equivalent electromagnetic potentials allows to detect the Aharonov-Bohm effect.

The proof of the main theorem consists of two steps. First, we introduce the broken (reflected) rays in each plane t = const, i.e. rays that undergo finite number of nontangential reflections at the obstacles. Using the geometric optics type solutions we prove that the gauge invariant boundary data uniquely determine the integrals of the magnetic (and subsequently electric) potentials over all broken rays. The second step of the proof is the study of the new branch

of X-ray tomography : the tomography of broken rays. We prove for some class of convex and piece-wise smooth obstacles in the plane that the integrals of magnetic potentials over broken rays uniquely determine the magnetic potential up to the gauge transformation (see details in arXiv:math.AP/0611342). In the end of the talk we shall describe results on the related inverse hyperbolic problems (c.f. arXiv:math.AP/0508161v2).