Inverse source problem in an advection dispersion reaction system

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Abstract.
Part I (work in collaboration with A. Hamdi). We consider the problem of identification of the unknown source $F(x,t) = \lambda(t)\delta(x-S)$ in the following system

$$\begin{array}{rcll}(\partial_t - D\partial_{xx} + V\partial_x + R)u(x,t) & = & F(x,t), & 0 < x < \ell, \;\; 0 < t < T \\ (\partial_t - D\partial_{xx} + V\partial_x + R)v(x,t) & = & Ru(x,t), & 0 < x < \ell, \;\; 0 < t < T\end{array}$$

from the data $[\{v(a,t), \partial_x v(a,t)\},\ \{v(b,t), \partial_x v(b,t)\}]$ where a and b are two points of $]0,\ell[$. Moreover, we suppose that one of them is strategic in a sense that will be described later. Assuming that the source F became inactive after the time T^* i.e. ($\lambda(t) = 0$ for $t \geq T^*$), we prove an identifiability result and propose an identification method.

Part II (work in collaboration with Du Duc Thang). Let us consider the operator

$$L[u] = u_t - D\Delta u + V \cdot \nabla u + Ru, \tag{0.1}$$

We consider the following problem

$$L[u] = \sum_{k=1}^{N} \gamma_k \rho(x - a_k), \quad x \in \mathbb{R}^2, t > 0, u(x,0) = 0, \tag{0.2}$$

where ρ is a given function, which belongs to $\mathcal{S}(\mathbb{R}^2)$. We denote the number of sources by N, the source positions by a_k, $k = 1, \ldots, N$ and the intensities γ_k. Assume that we have the measurement of u at some points b_j, $j = 1, \ldots, M$, i.e., $u(b_j, t) = d_j(t)$. We propose to determine uniquely the number of source N, the intensities and the source positions a_k with measurements on u at three point b_j.