A non-standard numerical method for variational data assimilation for a convection diffusion equation

Christian Clason
Zentrum Mathematik, TU München
clason@ma.tum.de

Abstract

Data assimilation problems are concerned with determining the initial condition at $t = 0$ in evolution equations from distributed or boundary measurements over a time interval $[0, T_0]$. These arise for example in weather or climate prediction, and are known to be ill-posed problems. The standard approach uses optimal control techniques for minimizing a suitable cost function on a linearized problem together with a regularization method and an optimality system making use of the adjoint state. The state in the time interval $[0, T_1]$, $T_1 > T_0$, is then calculated via a classical initial boundary value problem.

In this talk, we consider a linear convection diffusion equation and compute from distributed observations the state at the final time $t = T_0$, which can serve as an initial condition for the state equation in the time interval $[T_0, T_1]$. This "forward assimilation" problem is well-posed, as can be shown using an observability estimate for the adjoint state equation derived via a Carleman estimate. Its solution can be obtained via a series of null controllability problems for elements of a Hilbert basis of L^2, for which we present an efficient numerical method using proper orthogonal decomposition.

This is joint work with Jean-Pierre Puel (Université de Versailles, St. Quentin) and Peter Hepperger.