Higher-order topological sensitivity for elastodynamic inverse scattering

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To identify hidden obstacles from external measurements (e.g. overspecified boundary data) associated with the scattering of known incident waves, it is customary to invoke optimization algorithms. However, sensitivity to initial guesses, and also high computational costs for solving the forward problem in this context, have prompted the definition of so-called "sampling", or "probe", methods which aim at delineating in a computationally fast way the hidden obstacle(s), see e.g. the review paper [3]. Such techniques include the linear sampling [2], not pursued here, and the concept of topological sensitivity [1], whereby the sensitivity of a cost functional with respect to the creation of an infinitesimal object of small characteristic radius ε is quantified as a function of the small object location $\boldsymbol{x}_{\rm s}$. If $J(\varepsilon, \boldsymbol{x}_{\rm s})$ denotes the value achieved by the cost function used for solving the inverse problem when only the infinitesimal obstacle located at $\boldsymbol{x}_{\rm s}$ is present, then in 3D situations the topological derivative $\mathcal{T}_3(\boldsymbol{x}_{\rm s})$ associated with the nucleation of a small obstacle of volume $O(\varepsilon^3)$ and specified shape appears through the expansion $J(\varepsilon, \boldsymbol{x}_{\rm s}) - J(0, \boldsymbol{x}_{\rm s}) = \varepsilon^3 \mathcal{T}_3(\boldsymbol{x}_{\rm s}) + o(\varepsilon^3)$

In this communication, an extension of the topological derivative is presented, whereby $J(\varepsilon, \boldsymbol{x}_{\rm s})$ is expanded further in powers of ε. The expansion to order $O(\varepsilon^6)$ for 3D elastic scattering by a cavity, of the form $J(\varepsilon, \boldsymbol{x}_{\rm s}) - J(0, \boldsymbol{x}_{\rm s}) = \varepsilon_3^3 \mathcal{T}(\boldsymbol{x}_{\rm s}) + \varepsilon^4 \mathcal{T}_4(\boldsymbol{x}_{\rm s}) + \varepsilon^5 \mathcal{T}_5(\boldsymbol{x}_{\rm s}) + \varepsilon^6 \mathcal{T}_6(\boldsymbol{x}_{\rm s}) + o(\varepsilon^6)$, is considered, the order $O(\varepsilon^6)$ being important for cost functions J of least-squares format because the perturbations of the residuals featured in J are of order $O(\varepsilon^3)$ under the present conditions. General expressions for the coefficients, suitable for numerical computations, are given. In particular, for any *centrally-symmetric* infinitesimal cavity of characteristic radius ε centered at $\boldsymbol{x}_{\rm s}$, one finds that $\mathcal{T}_4(\boldsymbol{x}_{\rm s}) = 0$, while $\mathcal{T}_3(\boldsymbol{x}_{\rm s})$ is the so-called *topological derivative*, known from previous studies [1].

An approximate global search procedure can then be defined on the basis of the functions $\mathcal{T}_3(\boldsymbol{x}_{\rm s})\, \mathcal{T}_5(\boldsymbol{x}_{\rm s})$, $\mathcal{T}_6(\boldsymbol{x}_{\rm s})$. Such procedure and its attendant computational issues will be discussed. In particular, multipole expansions for elastodynamic fundamental solutions will be shown to substantially enhance its computational efficiency.

References

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