Multigrid algorithms for reconstruction problems with parabolic PDE constraints

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Abstract

We will present a multigrid algorithm for the solution of distributed parameter estimation problems with variable coefficients. The main feature of the method is that it is mesh-independent even in the case of zero regularization (when the data is in the range of the inversion operator). This makes the method algorithmically robust to the value of the regularization parameter. The method is based on a reduced space formulation in which we iterate in the inversion parameter space. We use a full multigrid scheme with a spectrally filtered stationary approximate-Hessian stationary smoother and standard intergrid transfer operators. We use Fourier analysis to estimate the overall performance of the scheme, and we provide numerical experiments that demonstrate the effectiveness of the method for different diffusion coefficients and regularization parameters. We have observed mesh-independent convergence factors resulting in O(N) complexity, where N is the number of state variables. We present and compare two smoothers, one based on simple spectral thresholding and one based on reduced Hessian approximation, based on inexact domain-decomposition approximations of the forward solvers. The latter approach is found to be robust, i.e., it results in a mesh independent convergence for all diffusion coefficient and regularization parameter values. Although the spectral filtering preconditioner has negligible computational cost it does not converge in all the cases unlike point-

wise preconditioner, e.g., it only works for large values of the regularization parameter. We also discuss the overall multigrid by using a 2-step iterative smoother, and Fourier transforms to do the intergrid transfers. We have shown that this scheme is extends to nonlinear reaction problems. Even in case of partial observations, these schemes result in near optimal algorithmic complexity.