A Family of Inertially Arbitrary Patterns that is not Potentially Nilpotent

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Abstract

An n by n sign pattern $\mathcal{S}$ is inertially arbitrary if each ordered triple (n_1, n_2, n_3) of nonnegative integers with $n_1 + n_2 + n_3 = n$ is the inertia of some matrix in $Q(\mathcal{S})$, the sign pattern class of $\mathcal{S}$. A new family of sign patterns $\mathcal{G}_n$ ($n \geq 4$) is presented that is proved to be inertially arbitrary for all odd $n = 2k + 1$. However, it is shown that $\mathcal{G}_{2k+1}$ is not potentially nilpotent and thus not spectrally arbitrary. To prove that $\mathcal{G}_{2k+1}$ allows each inertia with $n_3 \geq 1$, a novel method based on the Implicit Function Theorem is used, while matrices in $Q(\mathcal{G}_{2k+1})$ with inertias having $n_3 = 0$ are constructed by a recursive procedure from those of smaller order. Some properties of the coefficients of the characteristic polynomial of an arbitrary matrix having certain fixed inertias are derived, and are used to show that $\mathcal{G}_5$ and $\mathcal{G}_7$ are minimal inertially arbitrary sign patterns.