Information and Markets

William Zame

Summer School – PIMS/UBC

July 2006

Lecture 3: Mechanisms and Auctions

McLean-Postlewaite

- all cars same quality
- sellers know quality
- buyers know only distribution of quality
- distribution of quality uniform on $1, 4$
- #sellers ≥ 3

Efficient mechanism

- sellers report quality

- transfer automobiles at prices that depend on *all* reports

  - if (# H reports) $\geq \frac{1}{2}$ (#sellers): $t(H) = 4.5, t(L) = 4.4$

  - if (# H reports) $< \frac{1}{2}$ (#sellers): $t(H) = 1.4, t(L) = 1.5$

IRB, IRH, IRL, ICH, ICL ?

- everyone always makes strict gain

- no one ever gains by misrepresenting

Cautions

- truthful reporting is Nash equilibrium of mechanism:

  no *individual* can gain by misrepresenting

- mechanism may have *other* Nash equilibria

  that are *bad* from social point of view

- may not be possible to find mechanism whose equilibrium

  outcomes are *only good*

- may not be possible to support good outcome in strong

  Nash equilibria: no *coalition* can gain by misrepresenting

- coalition-proof equilibrium?

Exercises

- Is there an equilibrium of the game in which all sellers always report H?
- Is there an equilibrium of the game in which all sellers always lie?
- Find *all* the equilibria of this direct mechanism.

Seller information imperfect?

Assume

- sellers receive signal of true quality

	H	L
H	ρ	$1-\rho$
L	$1-\rho$	ρ

- $\rho > .5$ (signal is informative)

- signals independent conditional on true quality

For $\rho > .5$ same mechanism works if M large enough

- M large $\Rightarrow$ majority is nearly perfect predictor
- if misrepresentation does not change majority
  - misrepresentation gains $+.1$ or loses $-.1$
  - misrepresentation loses more often than gains
- if misrepresentation changes majority
  - may gain lot
  - unlikely

Another variant

$$u_s(H,m) = 4 + m \qquad u_s(L,m) = 1 + m$$
$$u_b(H,m) = 5 + m \qquad u_s(L,m) = 0 + m$$

Modification majority report = L $\longrightarrow$

- *do not* transfer automobile
- *do* make monetary transfers

Mechanism is *almost* efficient if #sellers large

Difference between Akerlof and McLean-Postlewaite environments?

- Akerlof: state = vector of qualities
- misreport *certain* to change perceived state
- McLean-Postlewaite: state = true quality
- misreport *unlikely* to change perceived state
- McLean-Postlewaite: agents are informationally small
- competition in information

McLean–Postlewaite If economy is large and agents are informationally small then there is an incentive compatible mechanism that achieves almost fully efficient outcomes.

Auctions

sealed bid second price auction

- bidders submit bids b_i

- high bid wins, pays 2nd-highest bid

equivalence with open outcry auction?

Private values

- 1 seller, 1 object
- N buyers
- v_i = valuation of i-th bidder
- $(v_1, \ldots, v_N) \in [0, 1]^N$
- joint probability distribution on $[0, 1]^N$

Weakly dominant strategy: $b(v) = v$

- bid determines whether or not win, not amount paid
- bid $x > v$: extra wins only when don't want object
- bid $x < v$: fewer wins only when do want object

Common values

- true value $= v \in [0,1]$

- cdf F, density f

$$\text{Prob}(v \leq x) = F(x) = \int_0^x f(s)ds$$

- signals $s_i \in [0,1]$, independent conditional on v

- cdf's $G(\cdot|\cdot)$, densities $g(\cdot|\cdot)$

$$\text{Prob}(s \leq x|v) = G(x|v) = \int_0^x g(s)\, ds$$

- monotone likelihood ratio:

$$\frac{\partial}{\partial v}\left(\frac{g(s|v)}{g(s'|v)}\right) \geq 0$$

  (high signal better news than low signal)

- g bounded

- smoothness, technical assumptions

Bidding strategy?

$$b(s) = E(v|s)$$

Wrong strategy ignores the fact that $b(s)$ wins because

s was highest signal and highest signal is overestimate of v

WINNER'S CURSE

Should condition on $b(s)$ winning $=$ s being highest signal

There is a unique symmetric equilibrium; bidding strategy is

$$b_i(s) = E(v|s_i = s \text{ and } s = \text{highest signal among others})$$

Information aggregation?

Auctions with N_r bidders, $N_r \to \infty$

price $p_r =$ second highest bid (random variable)

$$\lim_{r \to \infty} (p_r - v) \to 0 \quad \text{in probability ?}$$

NO

What is the highest bid? Calculus $\Rightarrow$

$$\max_s b(s) = E(v| \text{ one signal } = 1\} < v$$

independent of N

auctions for k identical objects

- bidders submit bids b_i

- high k bids win, pays $k+1$st-highest bid

There is a unique symmetric equilibrium; bidding strategy is

$$b_i^k(s) = E(v|s_i = s \text{ and } s = k\text{-th highest signal among others})$$

Winner's curse Bidding b and *winning* has a negative implication:

at most k others were led to bid *above* b.

Loser's curse Bidding b and *losing* has a positive implication:

at most $N - k$ others were led to bid *below* b.

Theorem (Pesendorfer & Swinkels)

For a sequence of auctions with k_r objects and N_r bidders

$$(p_r - v) \quad \longrightarrow \quad 0$$

$$\Updownarrow$$

$$k_r \to \infty \quad \text{and} \quad N_r - k_r \to \infty$$

- why must $k_r \to \infty$?
  - for k fixed

$$\max_s b(s) = E(v|k \text{ signals } = 1\} < v$$

    independent of N

- why is it OK that $N_r >> k_r$?
  - if $N_r >> k_r$ then most bidders submit tiny bids

Similar intuitions for

- voting (Fedderson & Pesendorfer)

- auctions of assets (Kremer)

- double auctions (Perry & Reny)