

# Quasicrystals: An Overview

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## **Metallic Phase with Long-Range Orientational Order and No Translational Symmetry**

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and

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*Centre d'Etudes de Chimie Métallurgique, Centre National de la Recherche Scientifique, F-94400 Vitry, France*

and

J. W. Cahn

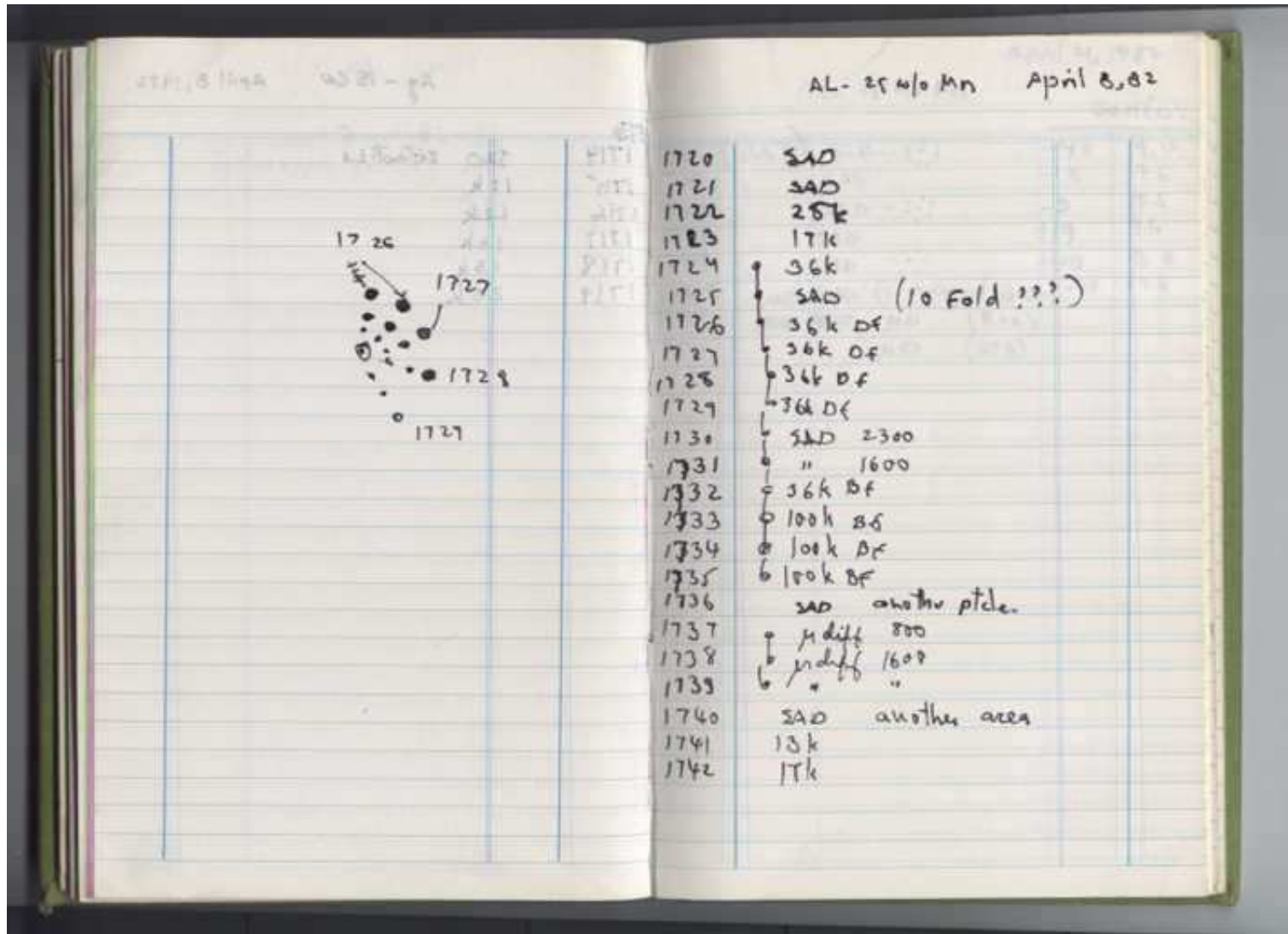
*Center for Materials Science, National Bureau of Standards, Gaithersburg, Maryland 20760*

(Received 9 October 1984)

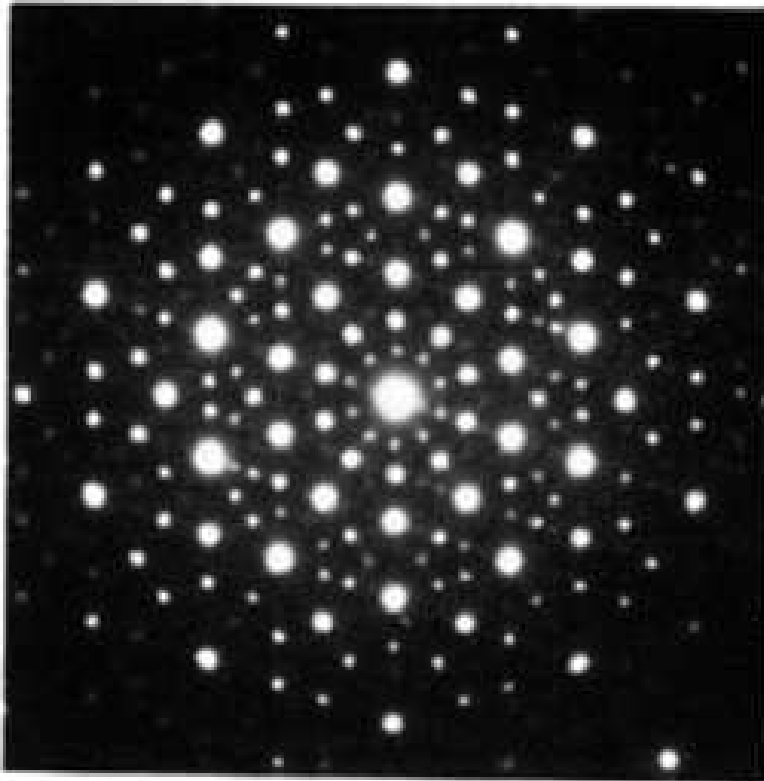
We have observed a metallic solid (Al–14-at.-%-Mn) with long-range orientational order, but with icosahedral point group symmetry, which is inconsistent with lattice translations. Its diffraction spots are as sharp as those of crystals but cannot be indexed to any Bravais lattice. The solid is metastable and forms from the melt by a first-order transition.

PACS numbers: 61.50.Em, 61.55.Hg, 64.70.Ew

# Shechtman's Experiment



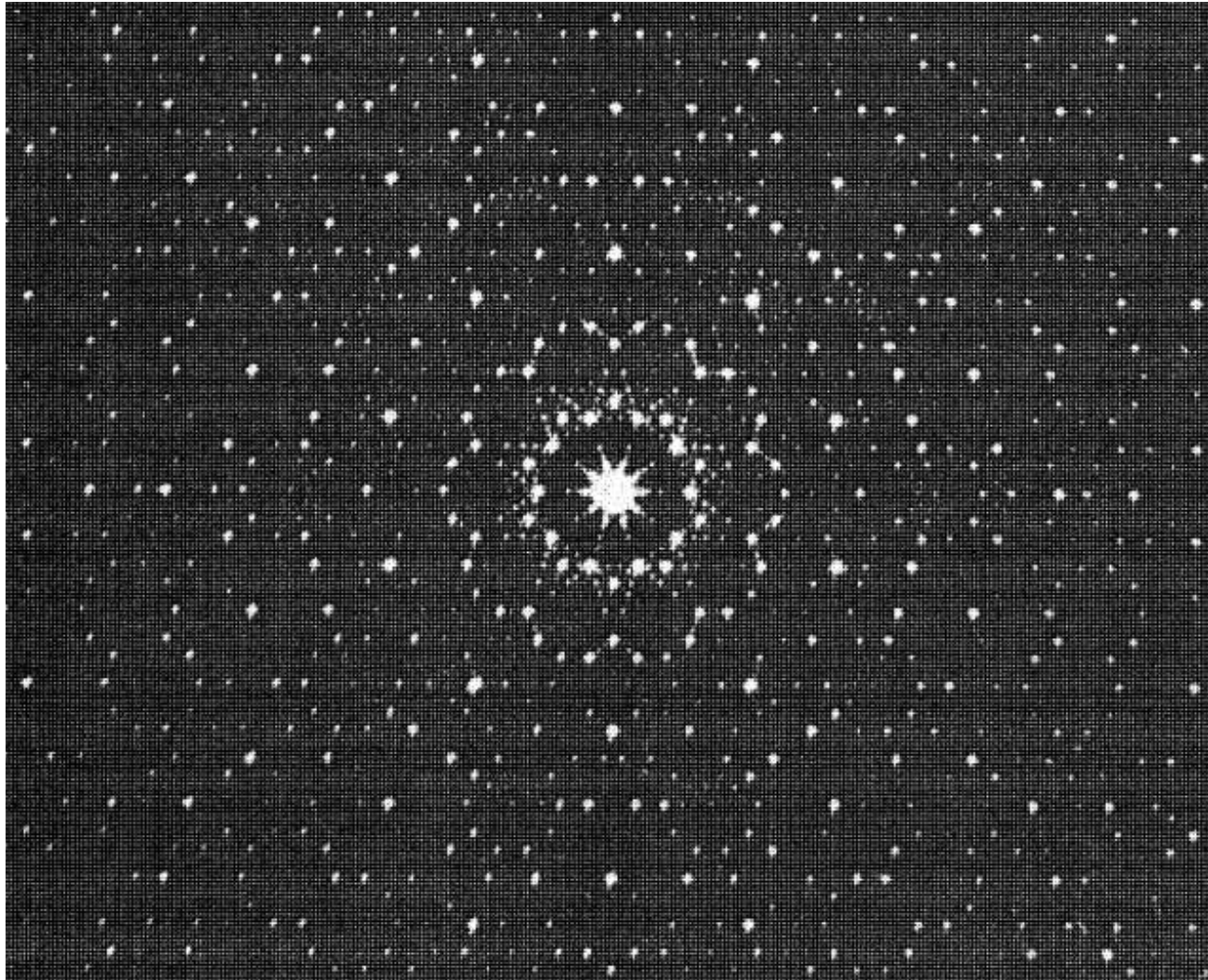
# Sharp Diffraction Peaks, 10-fold Symmetry



- electron diffraction pattern
- rapidly quenched Al-Mn alloy
- sharp diffraction spots, like a crystal
- 10-fold symmetry, so can't be a crystal!
- what can this be???

Twinned crystals can be ruled out by selected area diffraction and high resolution imaging.

# Diffraction Pattern of Penrose Tiling



A. Mackay, *Physica* **114A**, 609–613 (1982)



# Al-Mn Quasicrystal has Icosahedral Symmetry

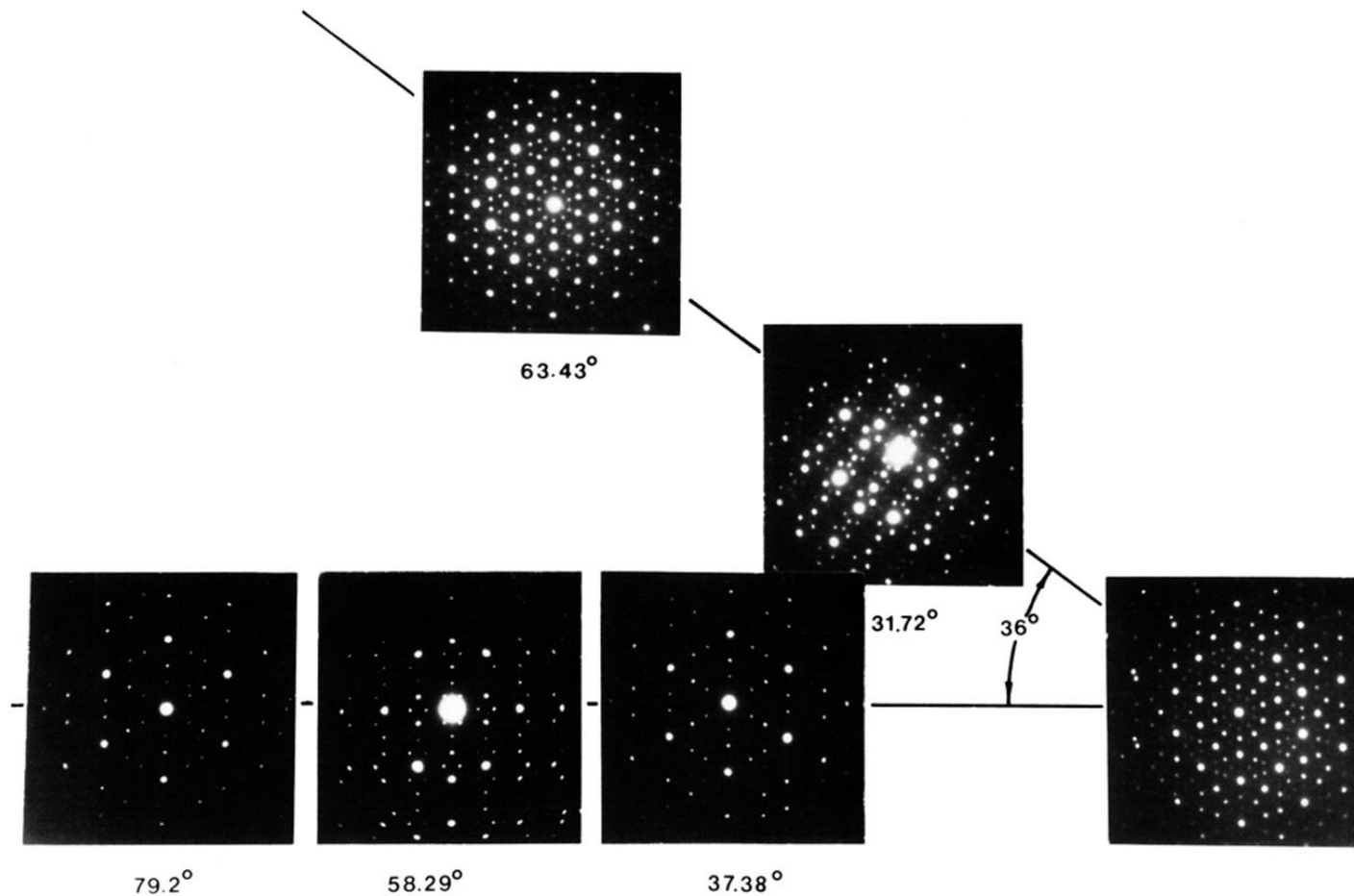


FIG. 2. Selected-area electron diffraction patterns taken from a single grain of the icosahedral phase. Rotations match those in Fig. 1.

# Today: Much Better Samples

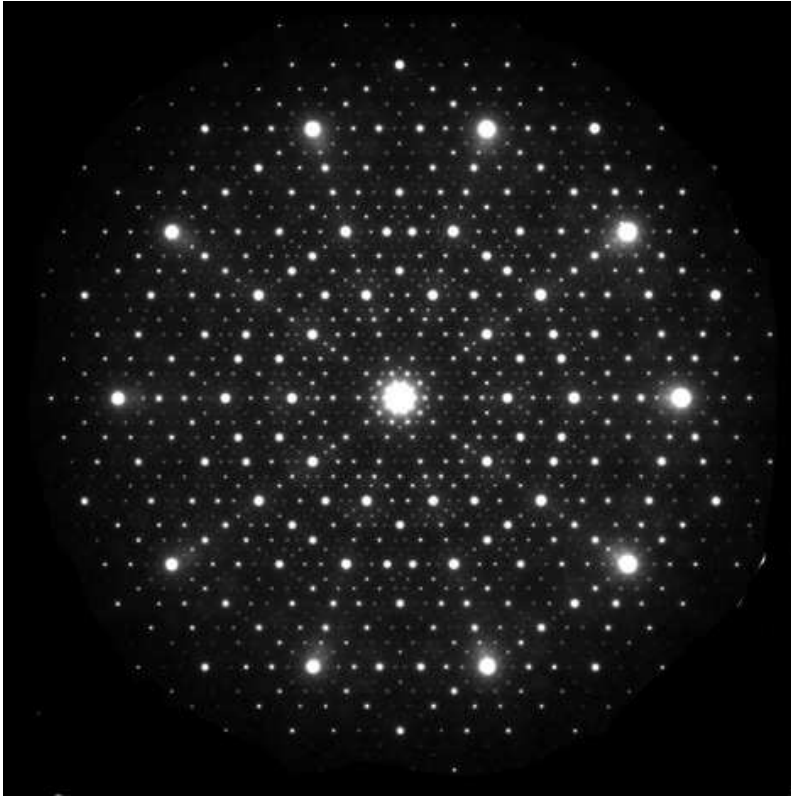


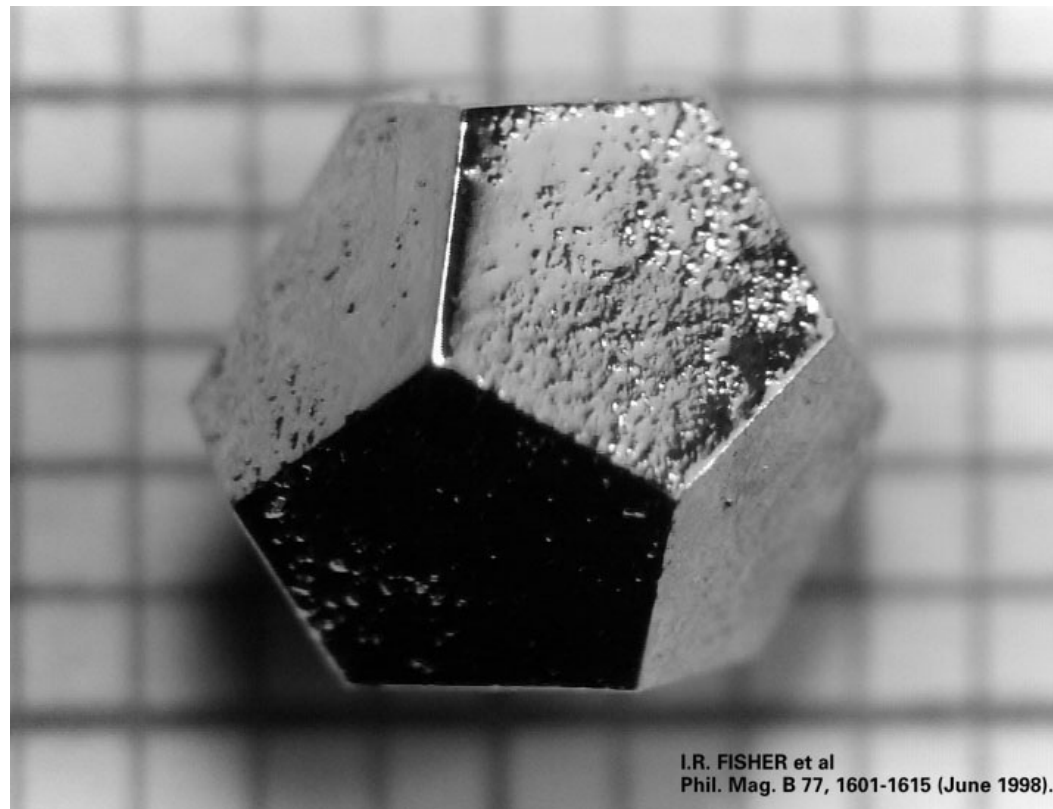
Image: C. Beeli

- several big families
- icosahedral Al-Pd-Mn, Al-Cu-Fe, Zn-Mg-RE, Cd-Ca, Cd-Yb, Zr-Ni-Ti, ...
- decagonal Al-Ni-Co, Al-Cu-Co, Al-Pd-Mn, ...
- octagonal and dodecagonal symmetry

Many quasicrystal phases are thermodynamically stable, at least at high temperature.

# Morphology

Often, mm- or even cm-sized single-quasicrystals can be grown.  
Non-crystallographic symmetry visible in morphology:





# Quasiperiodicity

Quasicrystals are **quasiperiodic**: density formally given by Fourier series

$$\rho(\mathbf{x}, \mathbf{x}^\perp) = \sum_{\mathbf{h} \in \mathbb{Z}^n} a_{\mathbf{h}} e^{i \sum_j h_j (\mathbf{k}_j + \mathbf{k}_j^\perp) \cdot (\mathbf{x} + \mathbf{x}^\perp)}$$

with  $n > d$  rationally independent basis vectors  $\mathbf{k}_j$

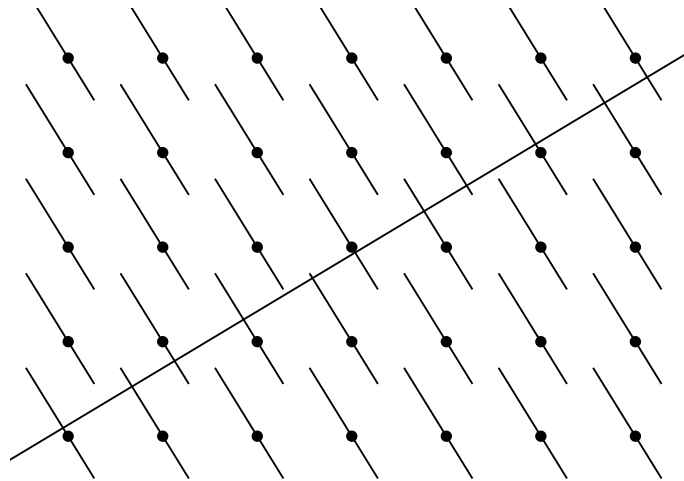
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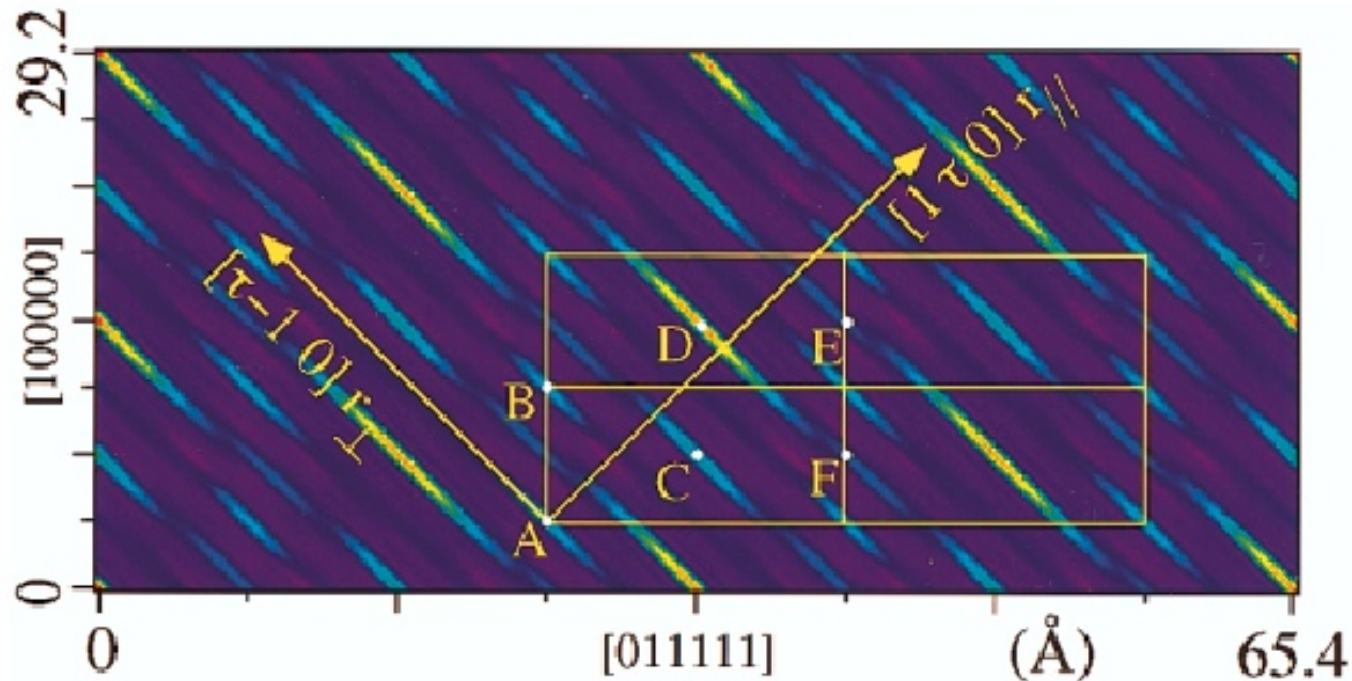
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In real space: cut through  $n$ -dimensional crystal:



# Structure Determination

Reconstructed electron density in 5-fold plane of i-ZnMgHo:



(H. Takakura et al., Phys. Rev. Lett. **86**, 236 (2001))

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Due to limited resolution, sharp boundaries of atomic surfaces are not measurable.

# Diffraction Techniques

## Neutron diffraction

- sees only nuclei
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## X-ray diffraction (also by synchrotron radiation)

- sees electron density, concentrated near heavy nuclei
- stronger coupling, but still weak enough for kinematic treatment
- main work horse for quantitative studies

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- tiny samples have to be used, but still multiple scattering (non-linear effects)
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Diffraction measures averaged structure, giving split positions, partial or mixed occupancies, ... Further modelling required!

# Imaging Techniques

High resolution Transmission Electron Microscopy:

- direct observation of structure and order
- sometimes difficult to interpret
- works best when looking along periodic direction (decagonal quasicrystals)

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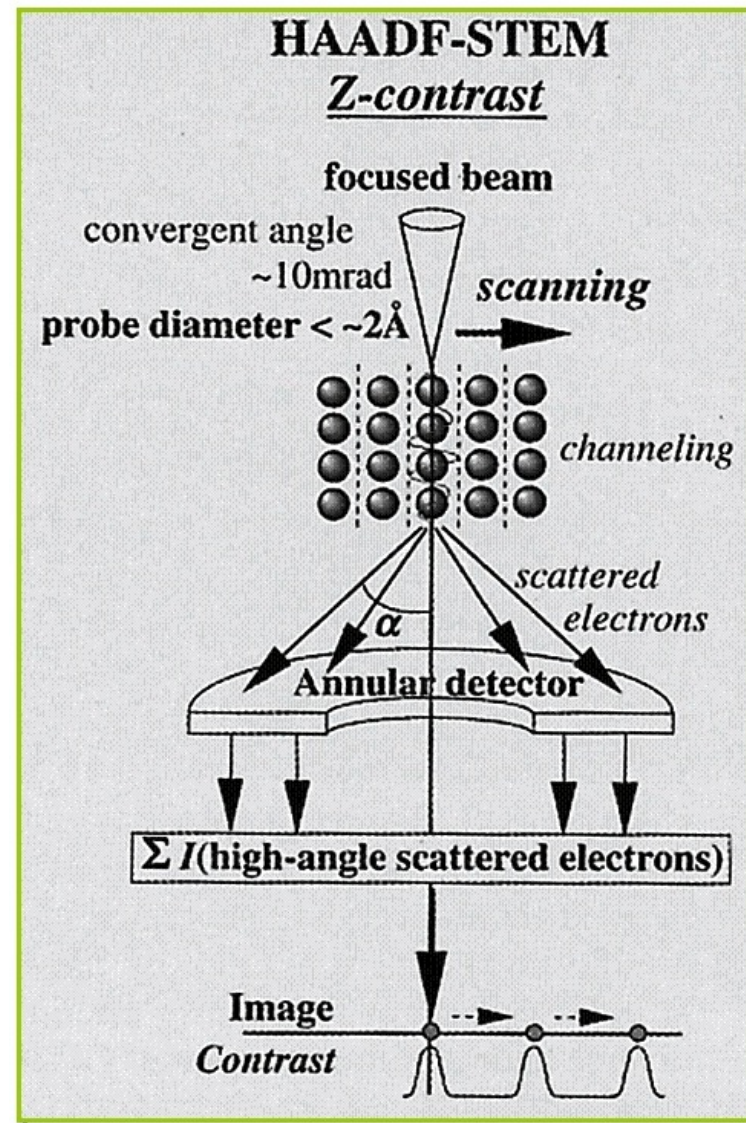
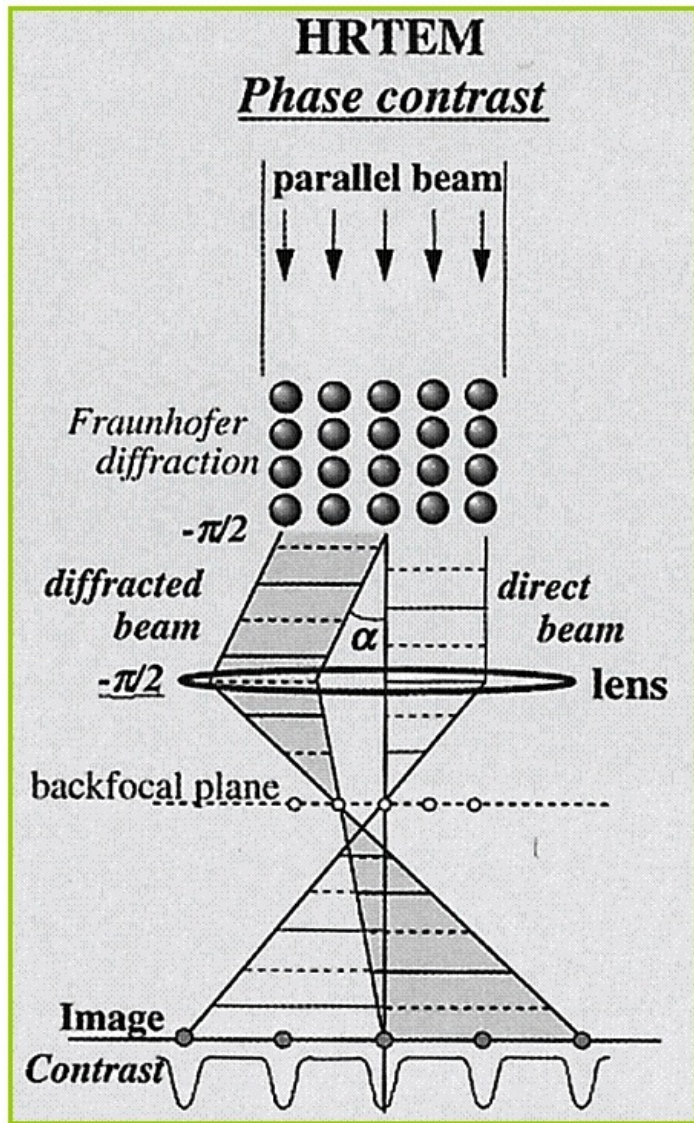
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Surface techniques:

- scanning tunneling microscopy
- atomic force microscopy
- works only at surfaces
- surface structure may differ from interior

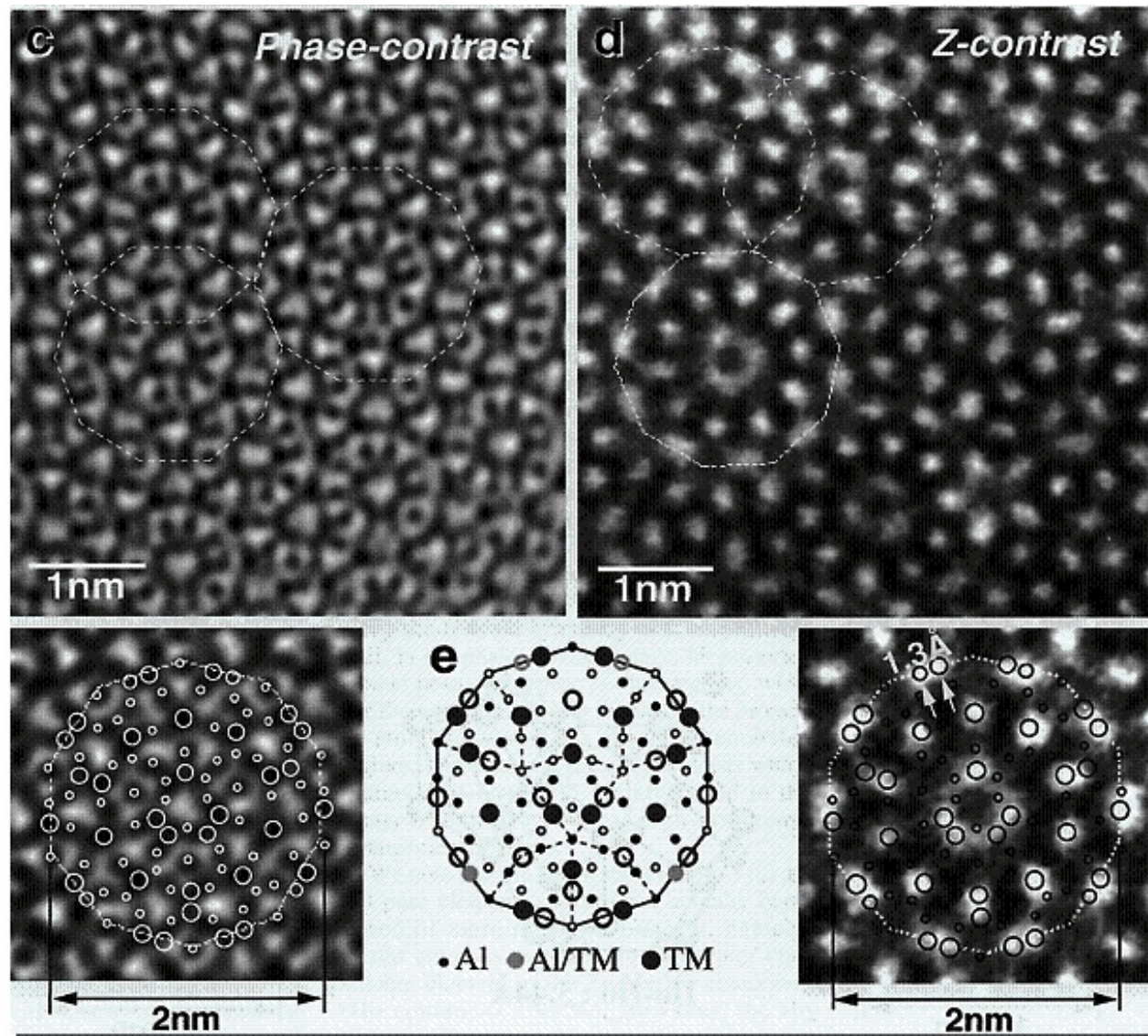
# HRTEM versus HAADF-STEM



Abe & Tsai, JEOL News 36 (2001) 18)



# Comparison HRTEM versus HAADF-STEM



Abe & Tsai, JEOL News 36 (2001) 18)



# Direct Structure Imaging

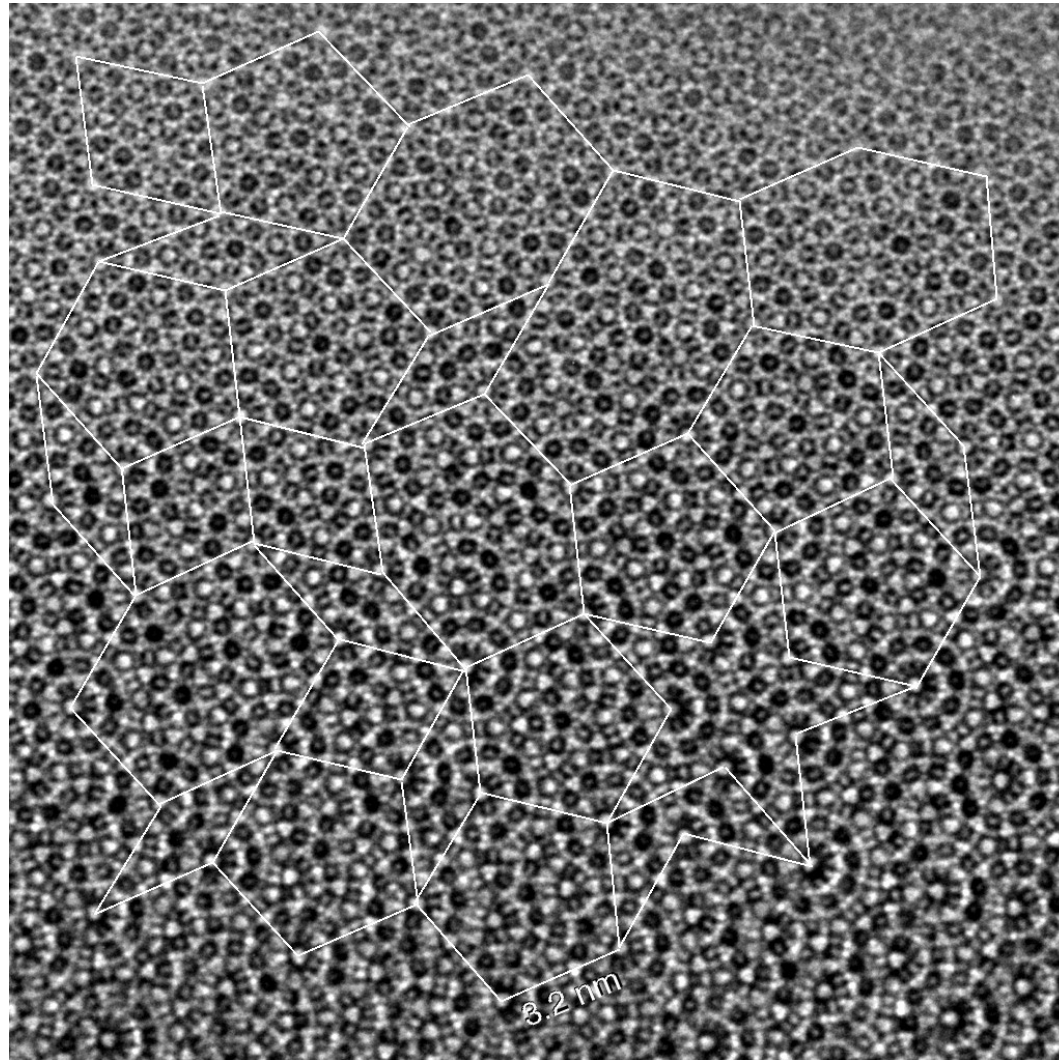
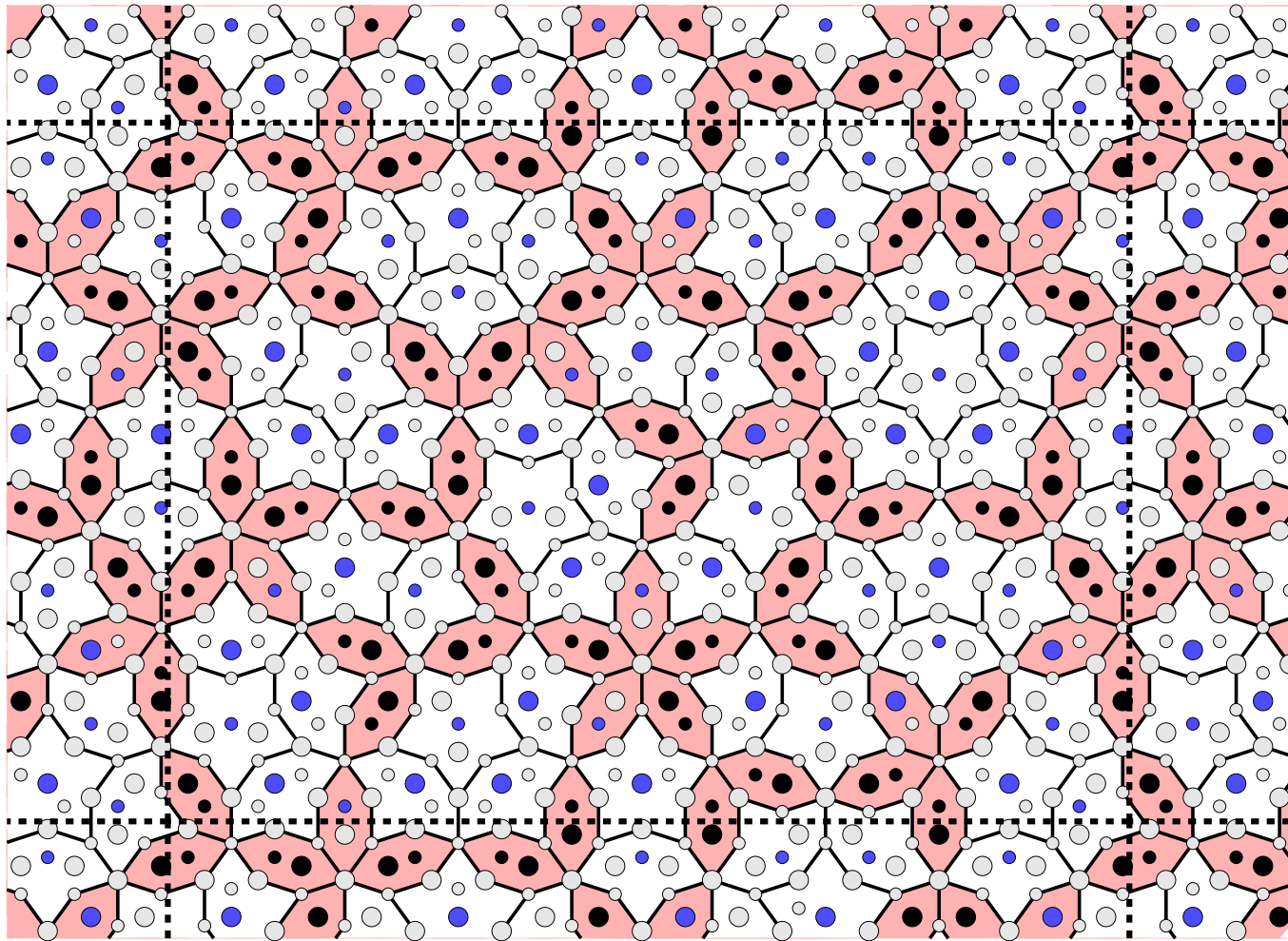


Image: S. Ritsch

# Quasicrystals as Decorated Tilings



M. Mihalkovic et al.

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Decorated tiling models are **highly desirable!**

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This allows to reshuffle tiles, play with structure variants, make periodic approximants.

Decorated tilings are an **idealization**, however.

Oftentimes, it is not clear what tiling to take.

Oftentimes, no tiling with **simple decoration** seems to exist.

# Total Energy Considerations

Diffraction experiments cannot determine all structure details.

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Compromise: use ab-initio derived classical potentials.