

Title: A class of self-dual partial differential equations and its variational principles

Abstract:

Motivated in part by the basic equations of quantum field theory (e.g. Yang-Mills, Chern-Simon, Seiberg-Witten and Ginzburg-Landau), we introduce and analyse a general --and remarkably encompassing-- concept of self (and antiself) dual partial differential equations. Besides the above equations, the class contains many of the basic families of linear and nonlinear, stationary and evolutionary partial differential equations: Transport equations, Nonlinear Laplace equations, Cauchy-Riemann systems, Navier-Stokes equations, Schrödinger equations, but also --infinite dimensional-- gradient flows of convex potentials (e.g., heat equations), Hamiltonian systems, and many other parabolic-elliptic equations. We then proceed to develop appropriate variational principles for a systematic resolution of such equations. In both stationary and dynamic cases, the equations associated to the proposed variational principles are not derived from the fact they are critical points of the action functional, but because they are also zeroes of certain derived non-negative Lagrangians.